(Ongoing research)

Towards Interferometric Lensless Endoscopy:

Rank-one Projections of Images Frequencies with Speckle Illuminations

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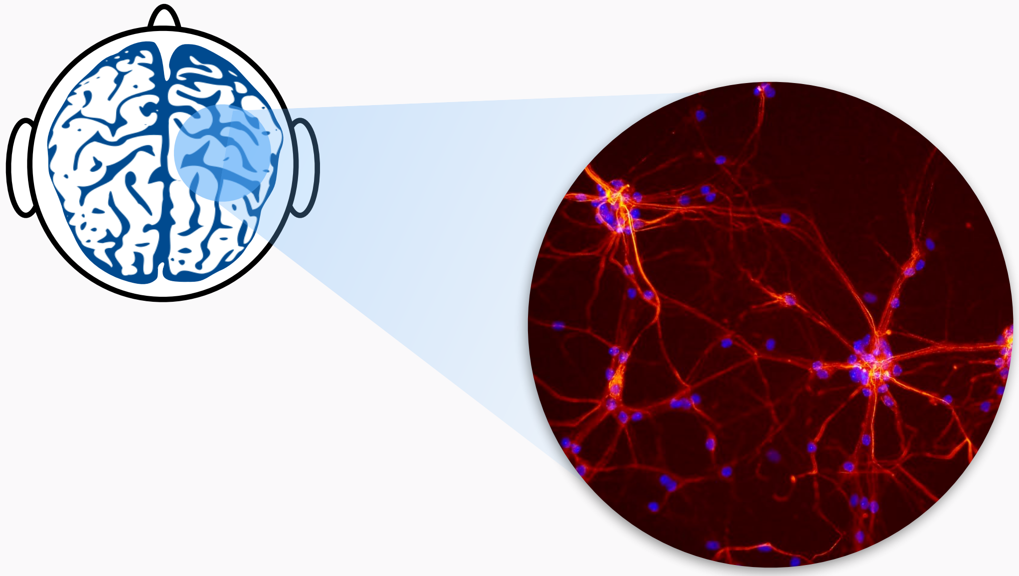
How to see neurons firing?



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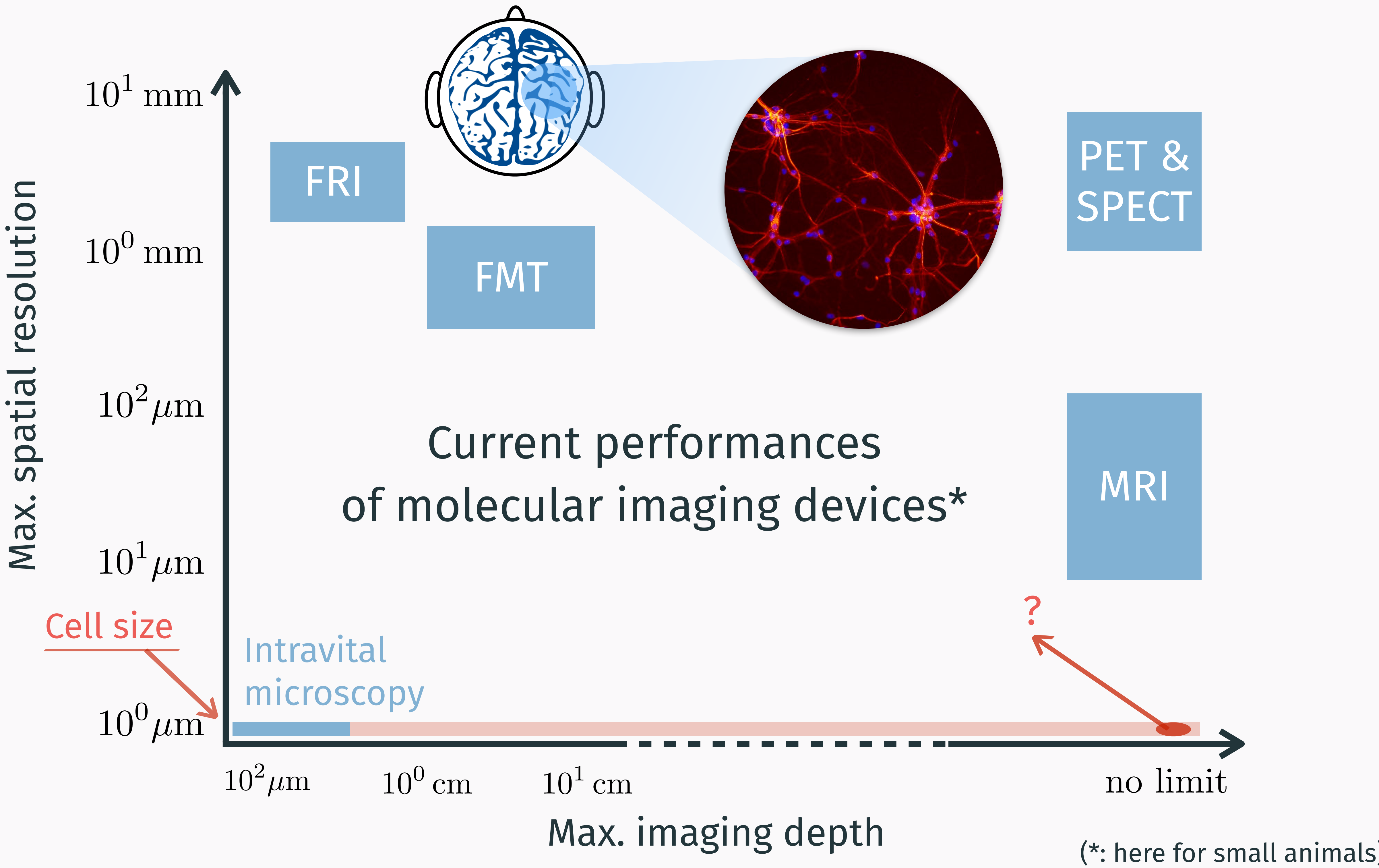

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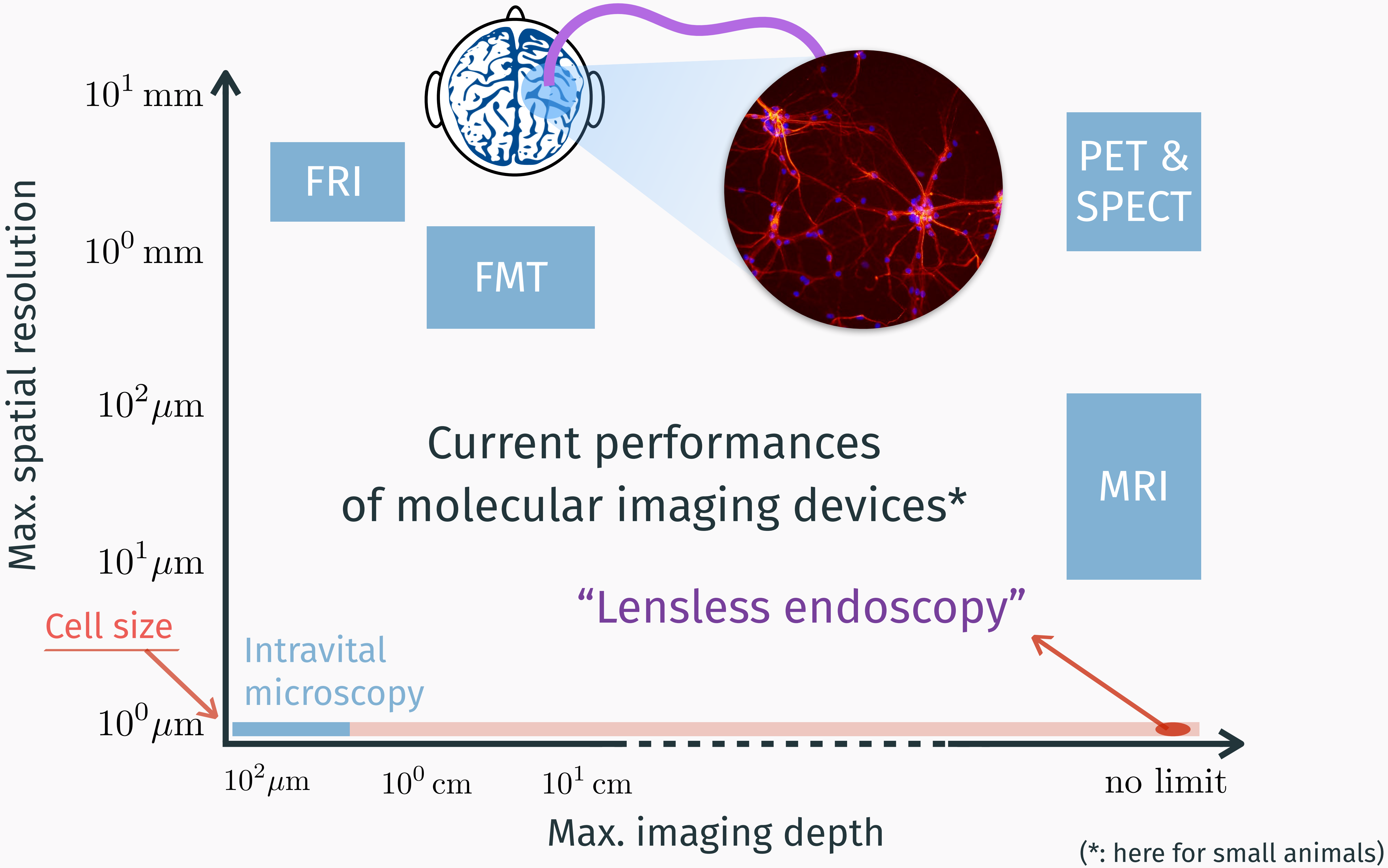

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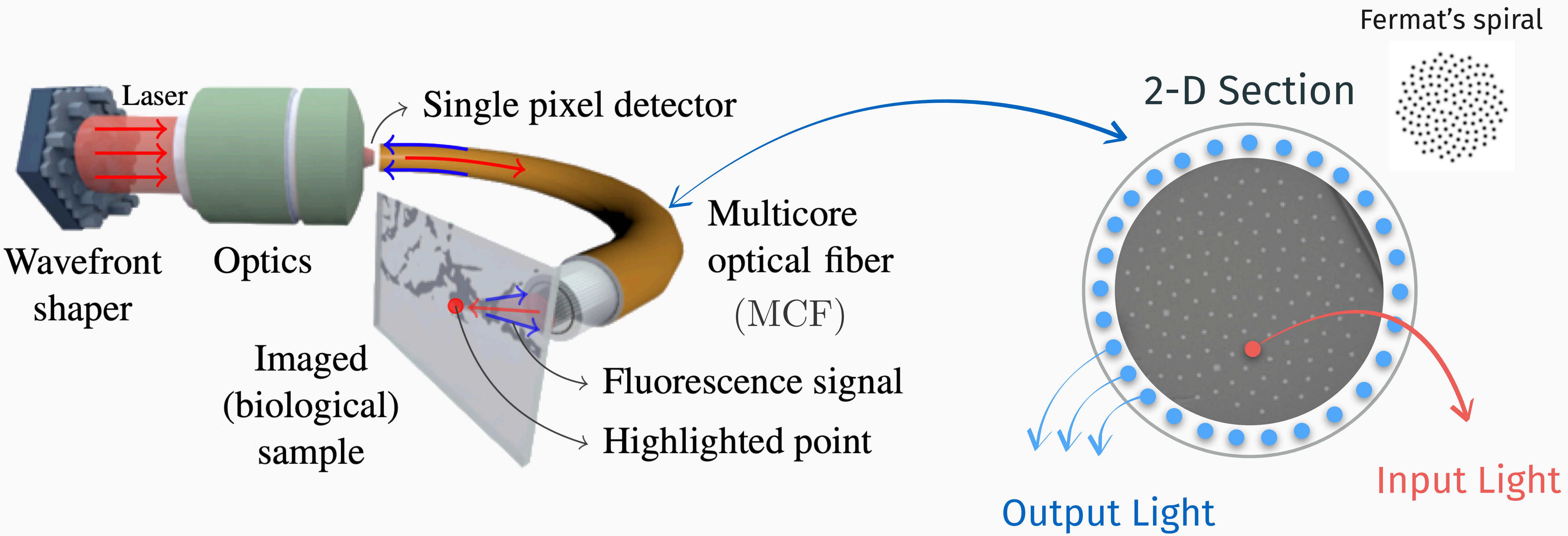
Lensless endoscopy: focused mode



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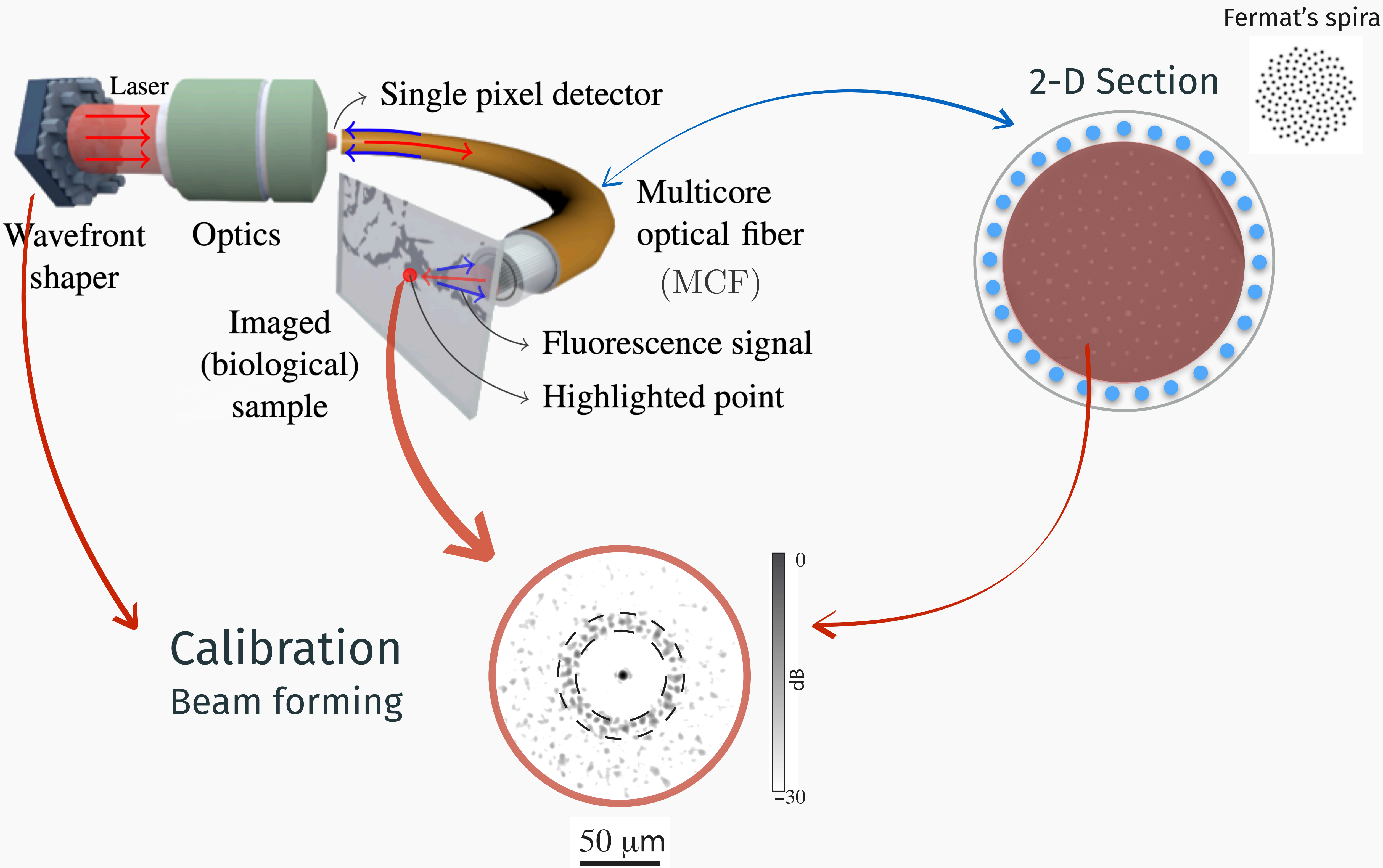

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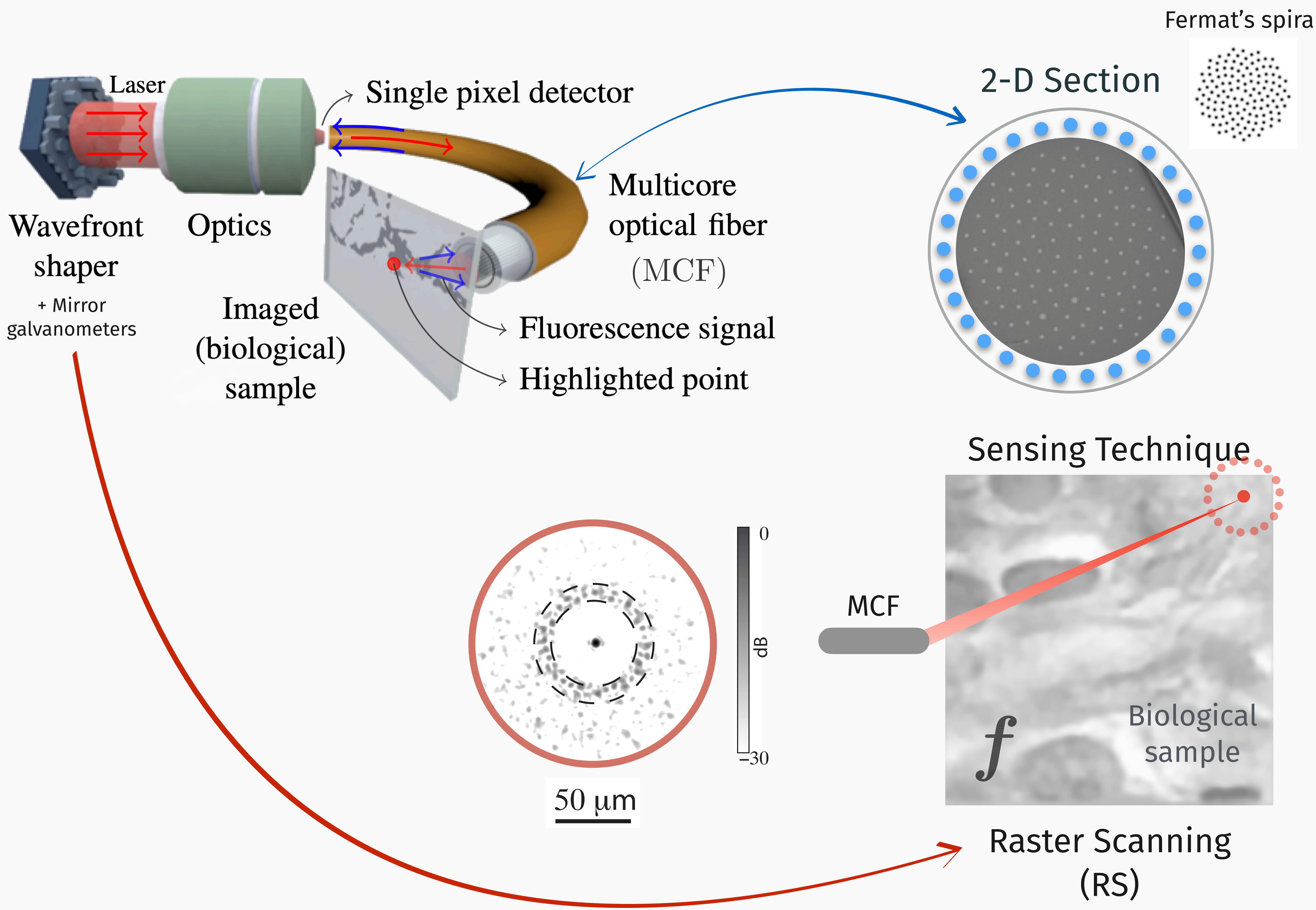

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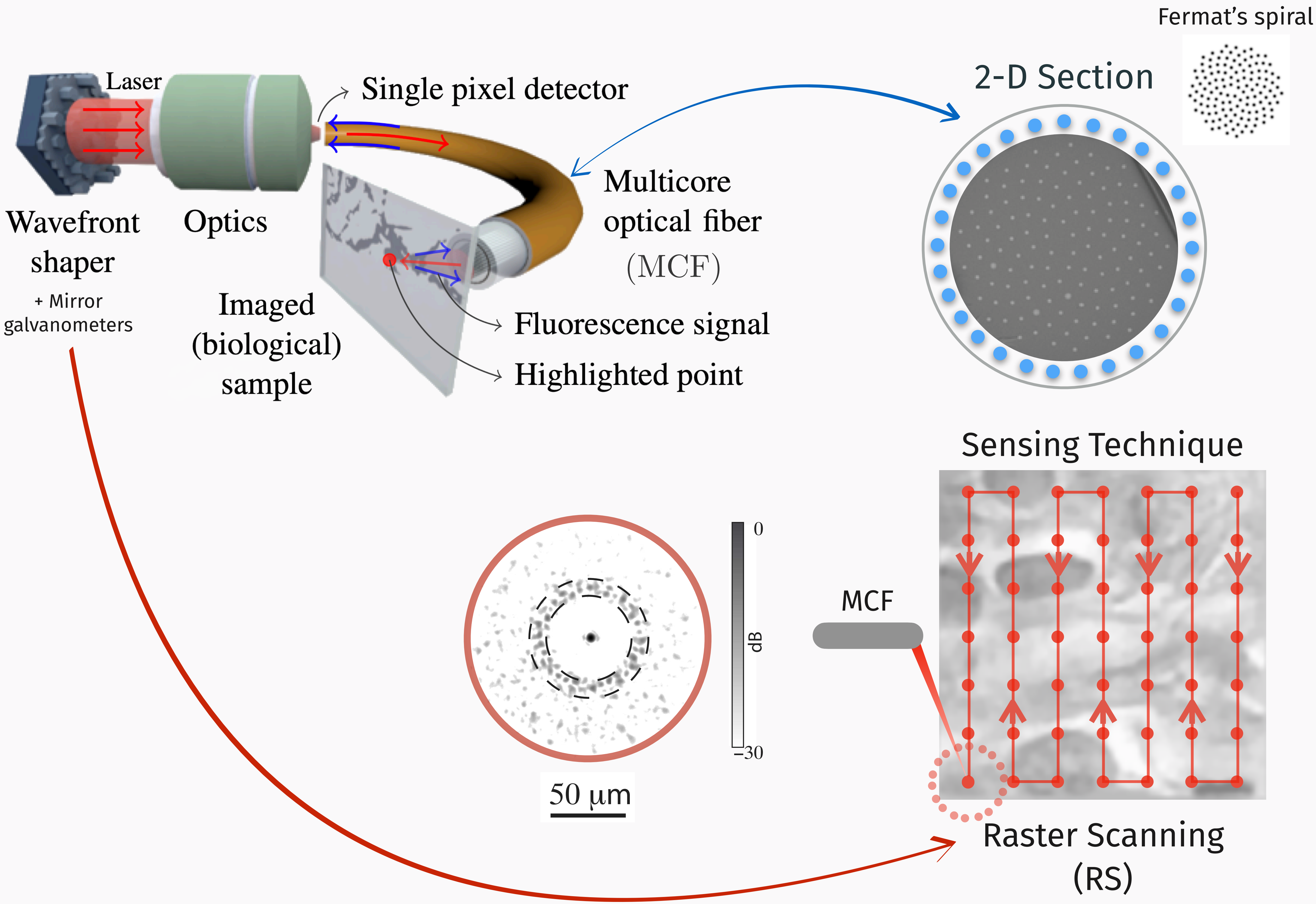

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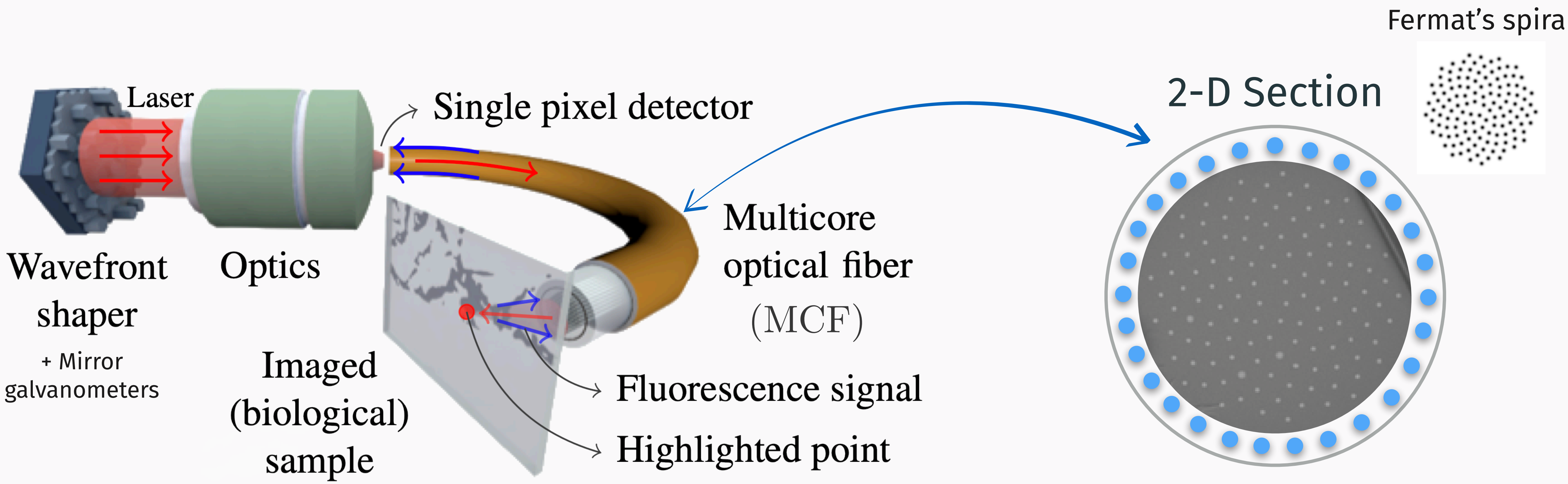
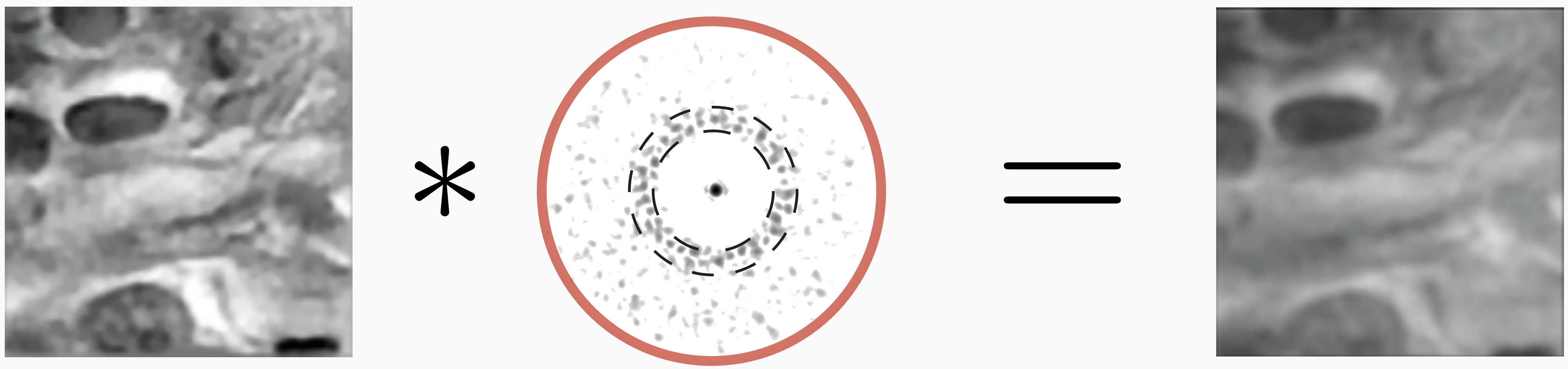


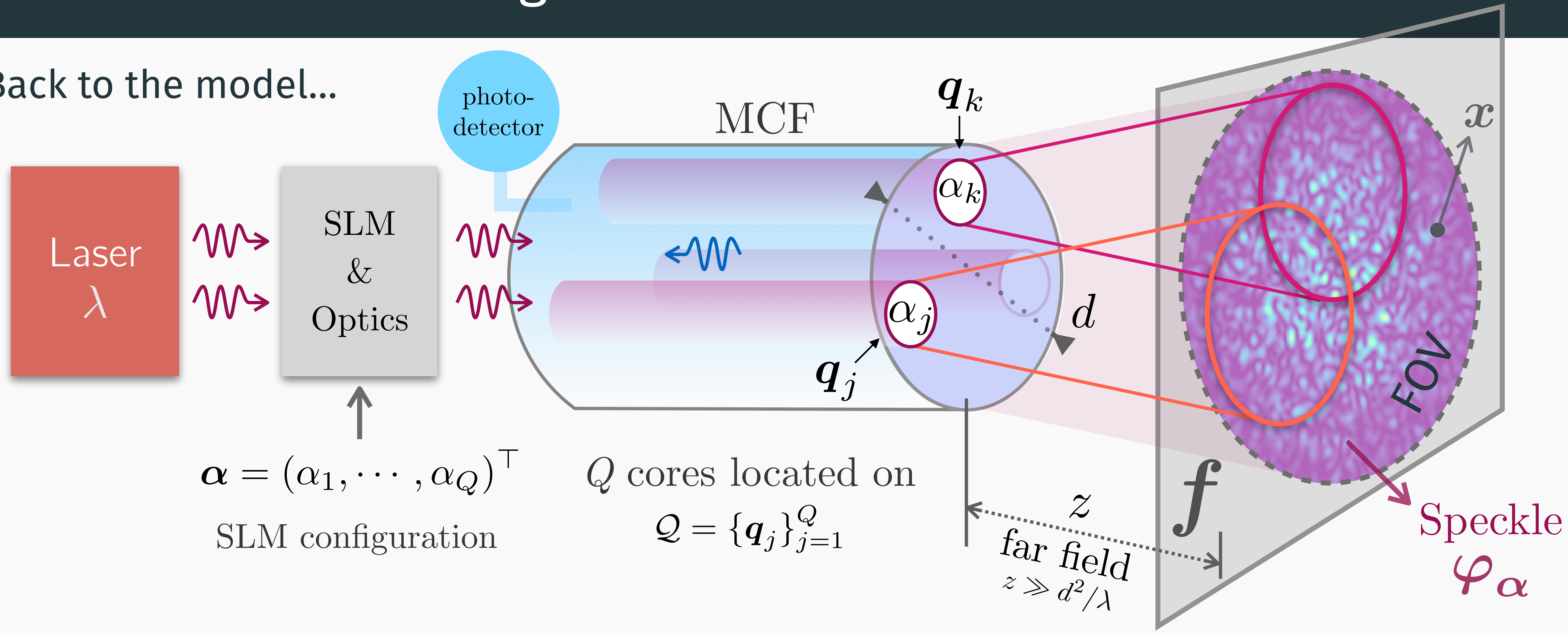


Sensing model

Direct Imaging

A closer look to sensing model

Back to the model...



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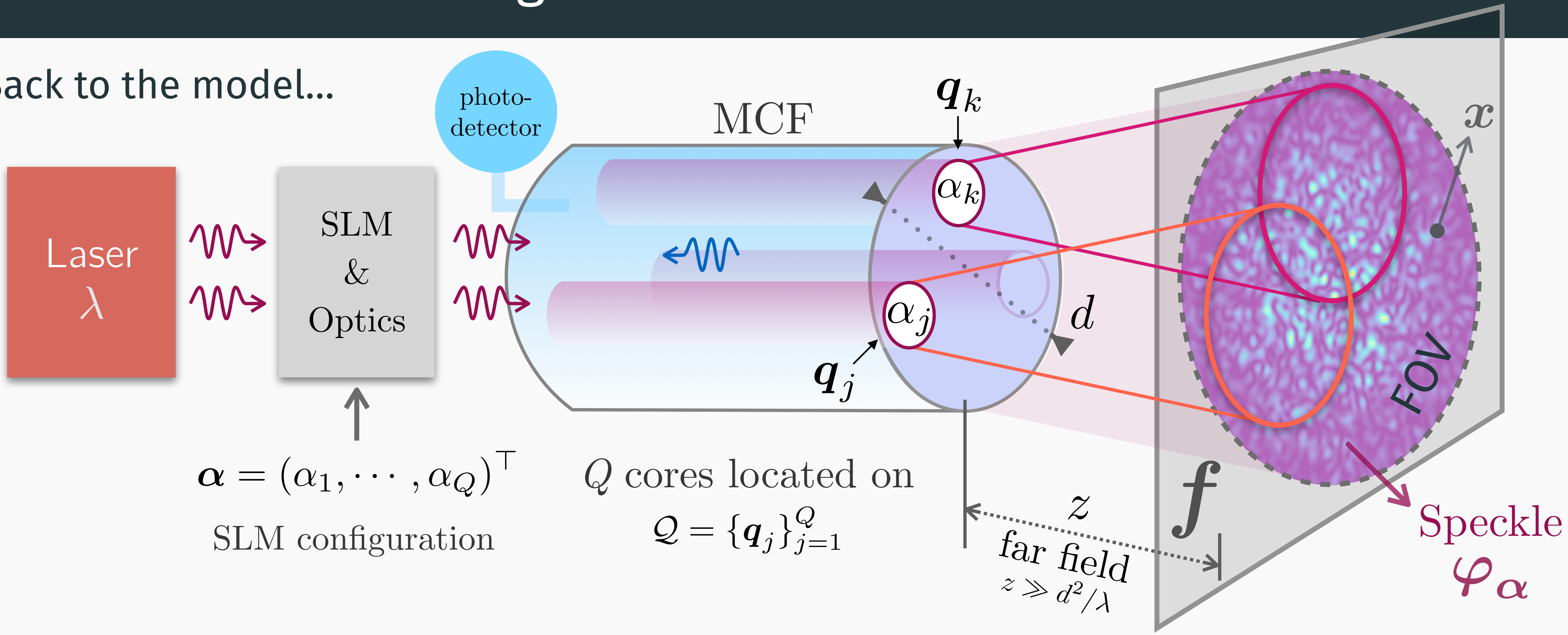

Speckles are interferences: (Under far-field approximation)

$$\varphi_{\boldsymbol{\alpha}}(\boldsymbol{x}) \propto \underbrace{w(\boldsymbol{x})}_{\text{FOV window}} \sum_{j,k=1}^{Q} \alpha_j \alpha_k^* e^{\frac{2\pi \mathrm{i}}{\lambda z}(\boldsymbol{q}_j - \boldsymbol{q}_k)^\top \boldsymbol{x}}$$

Why would this behave as a random Gaussian sensing?

Assuming $\varphi_{\boldsymbol{\alpha}}(\boldsymbol{x}) = w(\boldsymbol{x}) \sum_{j,k=1}^{Q} \alpha_j \alpha_k^* e^{\frac{2\pi \mathrm{i}}{\lambda z}(\boldsymbol{q}_j - \boldsymbol{q}_k)^\top \boldsymbol{x}}$,

$$
\begin{aligned}
y_{\boldsymbol{\alpha}} &= \langle \boldsymbol{\varphi}_{\boldsymbol{\alpha}}, \boldsymbol{f} \rangle = \textstyle\sum_{j,k=1}^{Q} \alpha_j \alpha_k^* \left[\int_{\mathbb{R}^2} e^{\frac{2\pi \mathrm{i}}{\lambda z}(\boldsymbol{q}_j - \boldsymbol{q}_k)^\top \boldsymbol{x}} w(\boldsymbol{x}) f(\boldsymbol{x}) \mathrm{d}\boldsymbol{x} \right], \\
&= \boldsymbol{\alpha}^* \boldsymbol{\mathcal{I}}[wf] \, \boldsymbol{\alpha},
\end{aligned}
$$

with the (Hermitian) *interferometric matrix* $\boldsymbol{\mathcal{I}}[wf] \in \mathbb{C}^{Q \times Q}$ s.t.

$$
(\boldsymbol{\mathcal{I}}[wf])_{j,k} := \int_{\mathbb{R}^2} e^{\frac{2\pi \mathrm{i}}{\lambda z}(\boldsymbol{q}_j - \boldsymbol{q}_k)^\top \boldsymbol{x}} \, w(\boldsymbol{x}) f(\boldsymbol{x}) \, \mathrm{d}\boldsymbol{x}.
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(noiseless) Interferometric sensing model (1/3)

Assuming $\varphi_{\boldsymbol{\alpha}}(\boldsymbol{x}) = w(\boldsymbol{x}) \sum_{j,k=1}^{Q} \alpha_j \alpha_k^* e^{\frac{2\pi \mathrm{i}}{\lambda z}(\boldsymbol{q}_j - \boldsymbol{q}_k)^\top \boldsymbol{x}}$,

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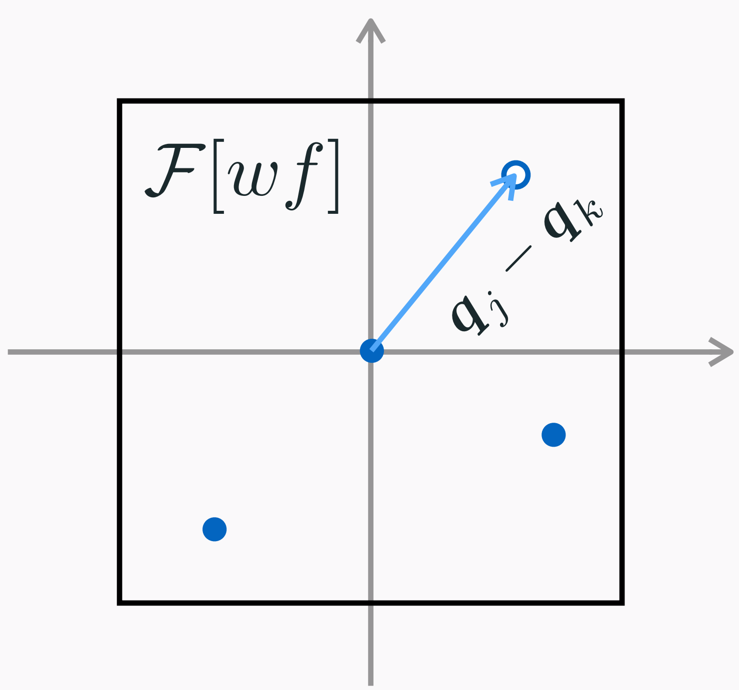
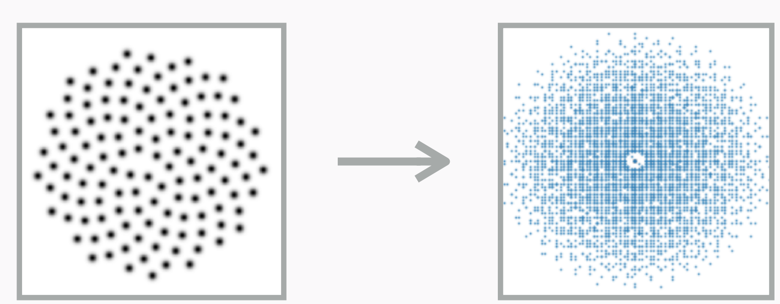


Observation: denser Fourier sampling if

$$
|\{\boldsymbol{q}_j - \boldsymbol{q}_k : \forall 1 \leq j, k \leq Q\}| \simeq Q^2
$$

- Lattices are bad core arrangements
- Fermat's spiral is not bad



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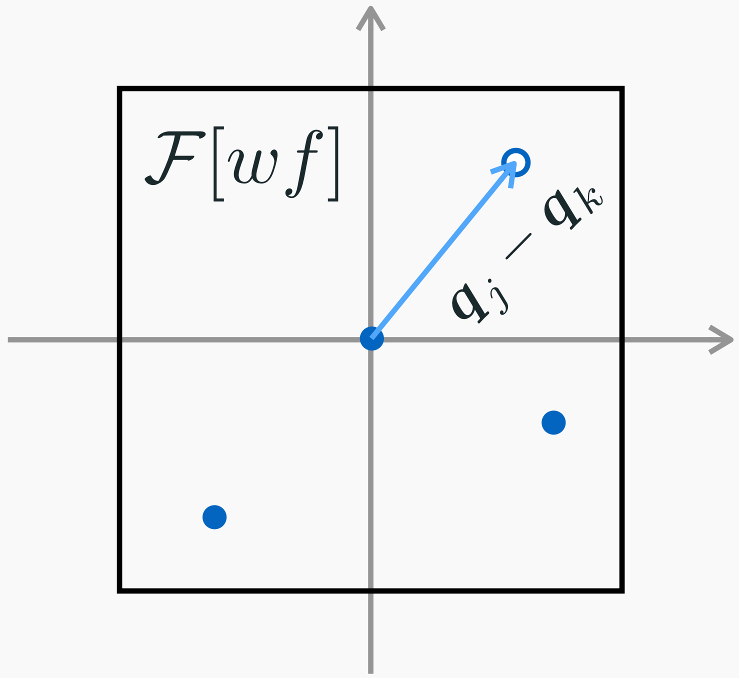
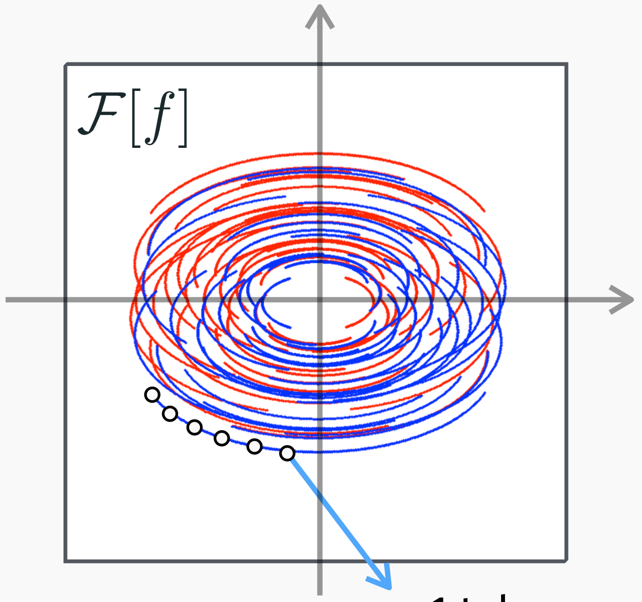


Similarity with radioastronomy!



General model:

$$\begin{aligned} y_{\boldsymbol{\alpha}} &= \boldsymbol{\alpha}^* \boldsymbol{\mathcal{I}}[wf]\, \boldsymbol{\alpha} + \text{noise}, \\ &= \operatorname{tr}(\boldsymbol{\alpha}\, \boldsymbol{\alpha}^* \boldsymbol{\mathcal{I}}[wf]) + \text{noise}, \\ &= \underbrace{\langle \boldsymbol{\alpha}\, \boldsymbol{\alpha}^*, \boldsymbol{\mathcal{I}}[wf] \rangle_{\mathrm{F}}}_{\substack{\text{Rank-one projection} \\ \text{of } \boldsymbol{\mathcal{I}}[wf]}} + \text{noise}, \end{aligned}$$

$$\boldsymbol{y} = (y_{\boldsymbol{\alpha}_1}, \cdots, y_{\boldsymbol{\alpha}_m})^\top = \mathcal{A}(\boldsymbol{\mathcal{I}}[wf]) := \{\langle \boldsymbol{\alpha}_j \boldsymbol{\alpha}_j^*, \boldsymbol{\mathcal{I}}[wf] \rangle_{\mathrm{F}}\}_{j=1}^m,$$

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with $\mathcal{A}(\boldsymbol{M}) := \{\langle \boldsymbol{\alpha}_j \boldsymbol{\alpha}_j^*, \boldsymbol{M}\rangle_{\mathrm{F}}\}_{j=1}^m$. (ROP)

Sample complexities of interest:

Does $\mathcal{A}$ capture enough from $\boldsymbol{\mathcal{I}}$? $\leftrightarrow$ m big enough?

Does $\boldsymbol{\mathcal{I}}$ capture enough from f? $\leftrightarrow$ Q big enough?

Core arrangement?

First answers from a few simplifications ...

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Simplifications

1. We consider a 1-D context: $\boldsymbol{q}_j \in \mathbb{R}^2 \to q_j \in \mathbb{R}$
2. Cores positioned on a regular grid: $q_j \in \{\Delta, 2\Delta, \cdots, N\Delta\}$

   $(\text{wlog, } \Delta = 1)$

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Interferometric sensing model: (Noiseless & Reloaded)

$$y_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}^* \boldsymbol{\mathcal{I}}[wf]\, \boldsymbol{\alpha} = \boldsymbol{\beta}^* \boldsymbol{\mathcal{J}}[wf]\, \boldsymbol{\beta}, \quad (\to \boldsymbol{y} = \mathcal{B}[\boldsymbol{\mathcal{J}}])$$

with $\boldsymbol{\beta} = \boldsymbol{\beta}(\boldsymbol{\alpha}) := \sum_{j=1}^{Q} \alpha_j \boldsymbol{e}_{q_j}$, $\{\boldsymbol{e}_k\}_{k=1}^{N} \subset \mathbb{R}^N$ canonical basis, and

$$(\boldsymbol{\mathcal{J}}[wf])_{j,k} := \int_{\mathbb{R}} e^{\frac{2\pi \mathrm{i}}{\lambda z}(j-k)x}\, w(x) f(x)\, \mathrm{d}x.$$

Extended interf. mtx

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Facts:

- $\triangleright$ $\boldsymbol{\mathcal{J}} \in \mathbb{C}^{N\times N}$ is Hermitian & Toeplitz, *i.e.* $(\boldsymbol{\mathcal{J}})_{j,k} = (\boldsymbol{\mathcal{J}})_{j-k} = (\boldsymbol{\mathcal{J}})^*_{k,j}$;
- $\triangleright$ if $f, w \geq 0$, then $\boldsymbol{\mathcal{J}}$ is PSD;
- $\triangleright$ if $f = \sum_{t=1}^r \gamma_t \delta(x - x_t)$ (with $\gamma_t > 0, x_t \in \mathbb{R}$), then $\boldsymbol{\mathcal{J}}$ has rank r, since

$$\boldsymbol{\mathcal{J}} = \sum_{t=1}^r \gamma_t w(x_t)\, \boldsymbol{u}\boldsymbol{u}^*; \quad \text{with } \boldsymbol{u} := \{e^{\frac{2\pi \mathrm{i}}{\lambda z}k}\}_{k=1}^N.$$

$\Rightarrow$ complexity of $\boldsymbol{\mathcal{J}}$ is r (not rN)

Possible 2-step reconstruction algorithm

1. Reconstruct $\mathcal{J}$ with [Chen, 15]

$$\hat{\mathcal{J}} = \underset{\boldsymbol{M} \in \mathbb{C}^{N \times N}}{\arg\min} \; \|\boldsymbol{M}\|_* \quad \text{s.t.} \quad \begin{aligned} &\boldsymbol{M} \succcurlyeq 0, \; \|\boldsymbol{y} - \mathcal{B}[\boldsymbol{M}]\|_2 \leq \varepsilon, \\ &\boldsymbol{M} \text{ is Toeplitz.} \end{aligned}$$

2. Unfold $\hat{\mathcal{J}}$ $\rightarrow$ get an estimate of the FT $\mathcal{F}[wf]$ of wf.
3. Inverse the FT and get wf. (at a resolution limited by N and $\triangle$)

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Theorem. [Chen '15] If $\boldsymbol{\beta}$ is sub-Gaussian (i.i.d β_i, $\|\beta_i\|_{\psi_2} = O(1)$), then, provided $\|\text{noise}\| \le \varepsilon$, and

$$m \ge C\, r \log^{10} N,$$

(Partial answer)

we have (uniformly over any rank-r $\mathcal{J}$)

$$\|\hat{\mathcal{J}} - \mathcal{J}\|_{\mathrm{F}} \le C \frac{\varepsilon}{\sqrt{m}}.$$

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Simulations (1-D)

In practice, we solve (with ADMM)

$$\hat{\boldsymbol{f}} = \arg\min_{\boldsymbol{f}'} \|\boldsymbol{f}'\|_1 \quad \text{s.t. } \|\boldsymbol{y} - \mathcal{A}[\boldsymbol{\mathcal{I}}[\boldsymbol{w} \odot \boldsymbol{f}']]\|_2 \leq \varepsilon,$$

Transition matrices (1-D case):

- r-sparse signals
- Random core arrangement
- Complex Gaussian ROP
- 60 trials
- Success if $\geq$ 40 dB

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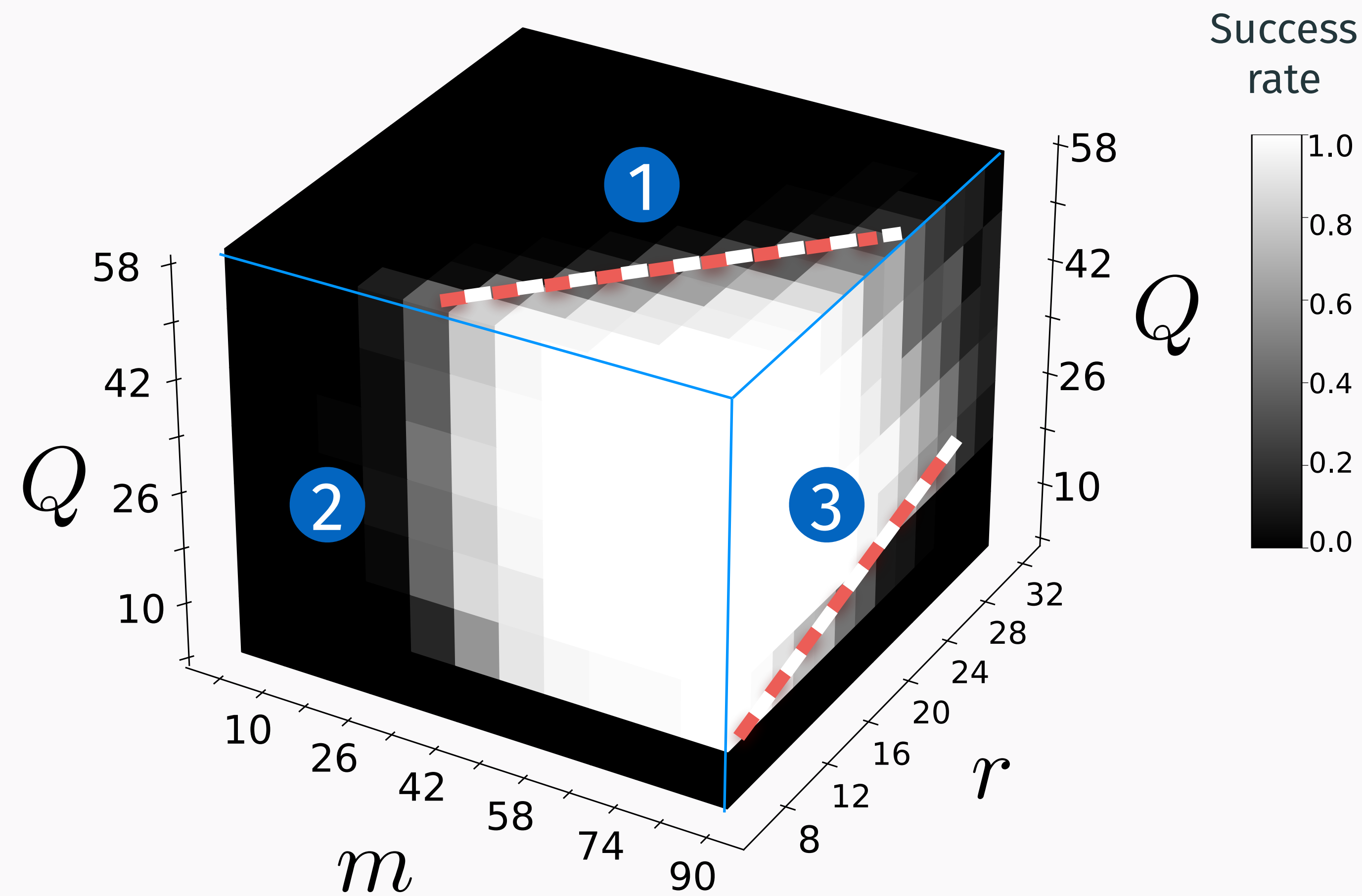
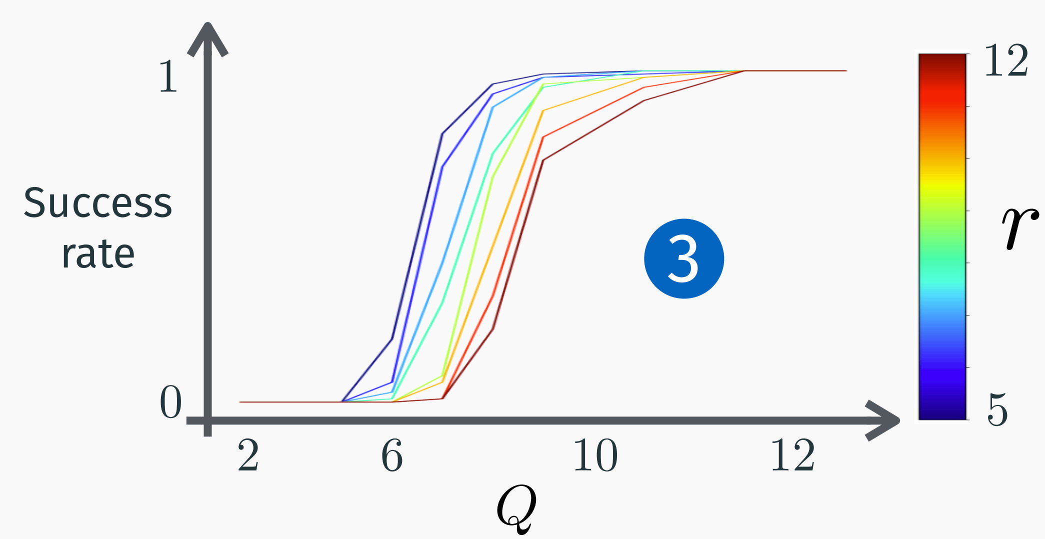


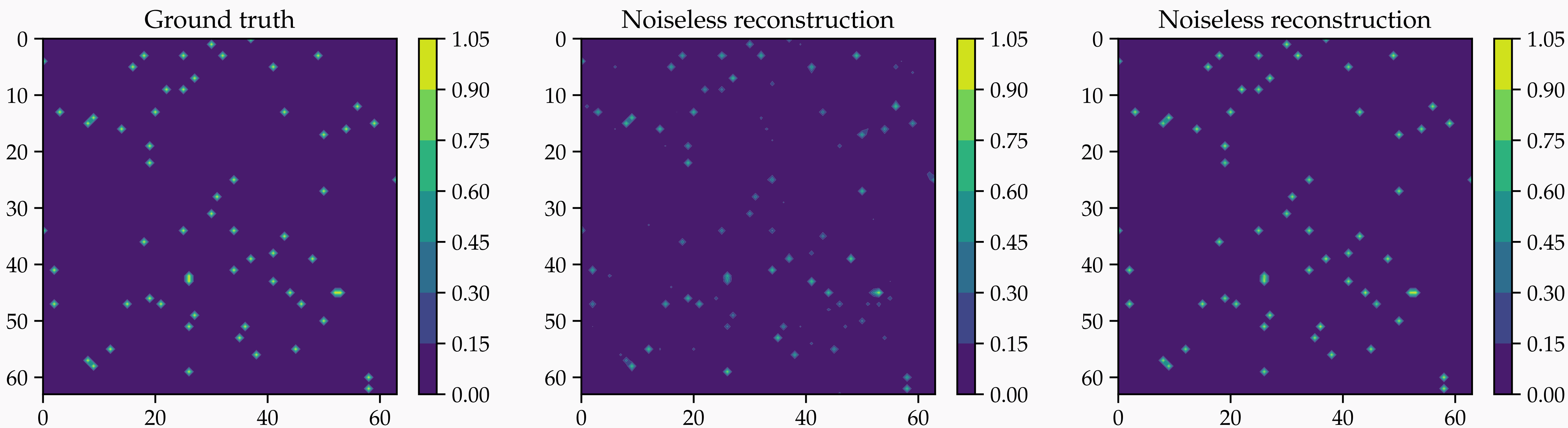
Observations:

1 : “$m \geq Cr$” if Q large

2 3: “$\min(m, Q) \geq Cr$”



Extending the reconstruction in 2-D. (preliminary results)



$m/r = 5 \quad m/N = 8\%$

$\mathrm{SNR} = 5.71\mathrm{dB}$

$m/r = 10 \quad m/N = 16\%$

$\mathrm{SNR} = 21.26\mathrm{dB}$

& SNR > 40dB for $m/r = 20$

Sensing parameters: $N = 4096 = 64 \times 64$, $Q = 100$.

Object parameter: $r = 64$ (= sparsity level)

Actual experiments are ongoing...

To conclude ...

Take away messages:

- Fluorescent compressive speckle imaging (with MCF) follows an interferometric sensing model;
- This model amounts to "rank-one projecting" an interferometric Toeplitz matrix $\mathcal{I}[wf]$;
- Sparse images lead to low-rank $\mathcal{I}[wf]$ that can be recovered.

Open questions:

- Theoretical guarantees for sparse random vectors $\boldsymbol{\alpha}$;
- Using more advanced sparsity models: *e.g.*, sparsity in levels + wavelets, weighted sparsity;
- Optimization of core arrangements.

Thank you for your attention

Related bibliography:

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