Interferometric Single-pixel Imaging with a Multicore Fiber





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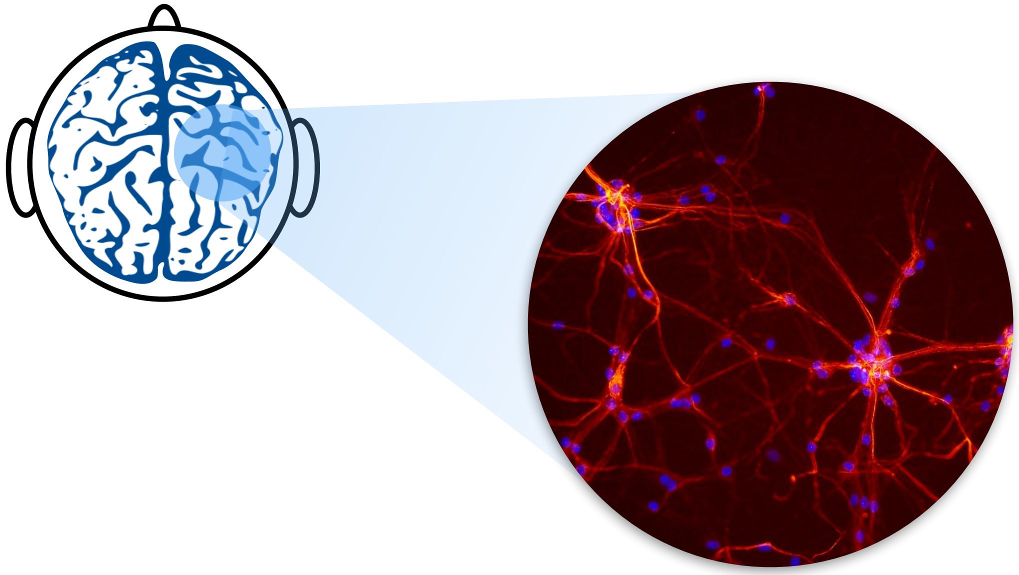
Luxembourg

How to see neurons firing?



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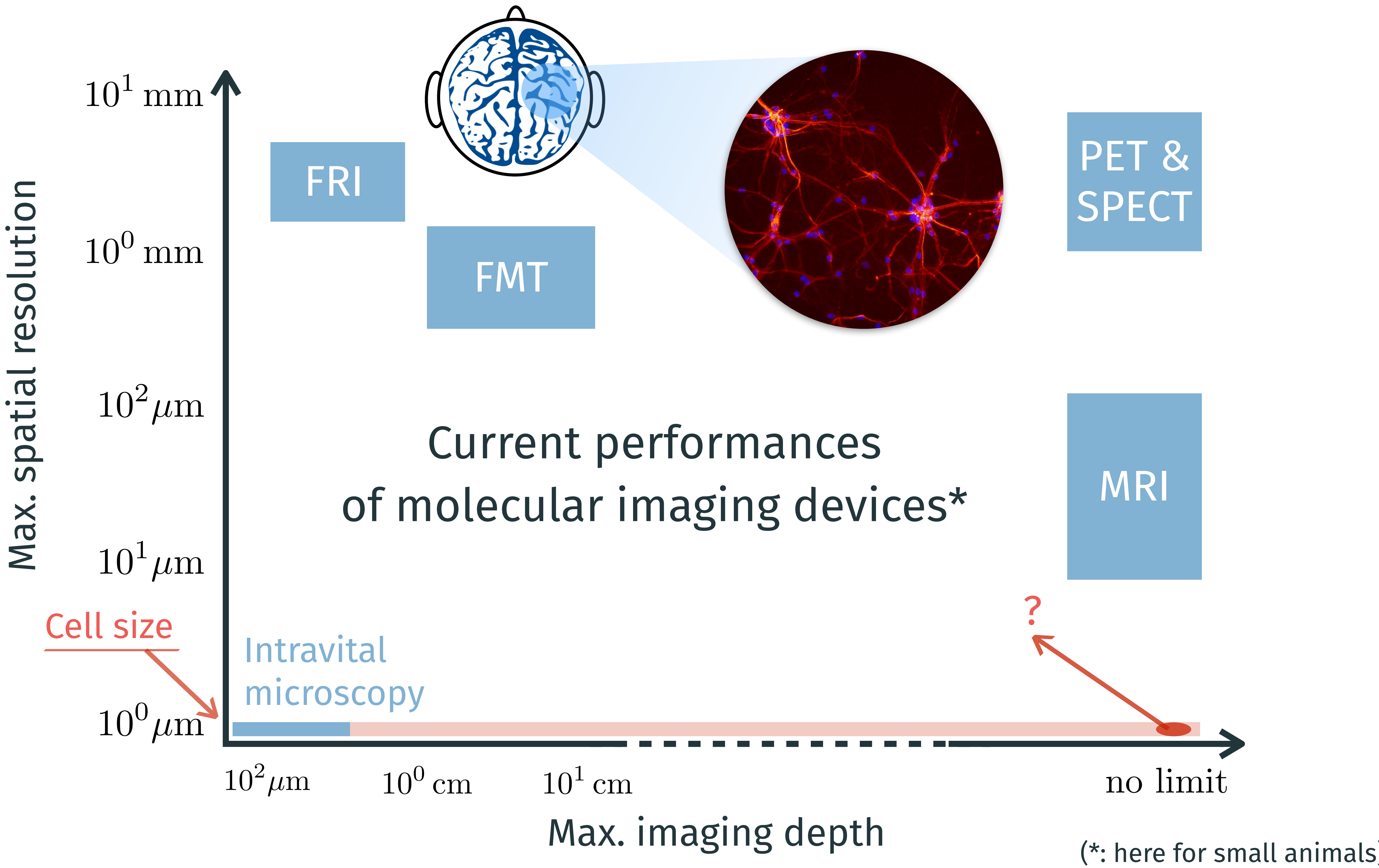

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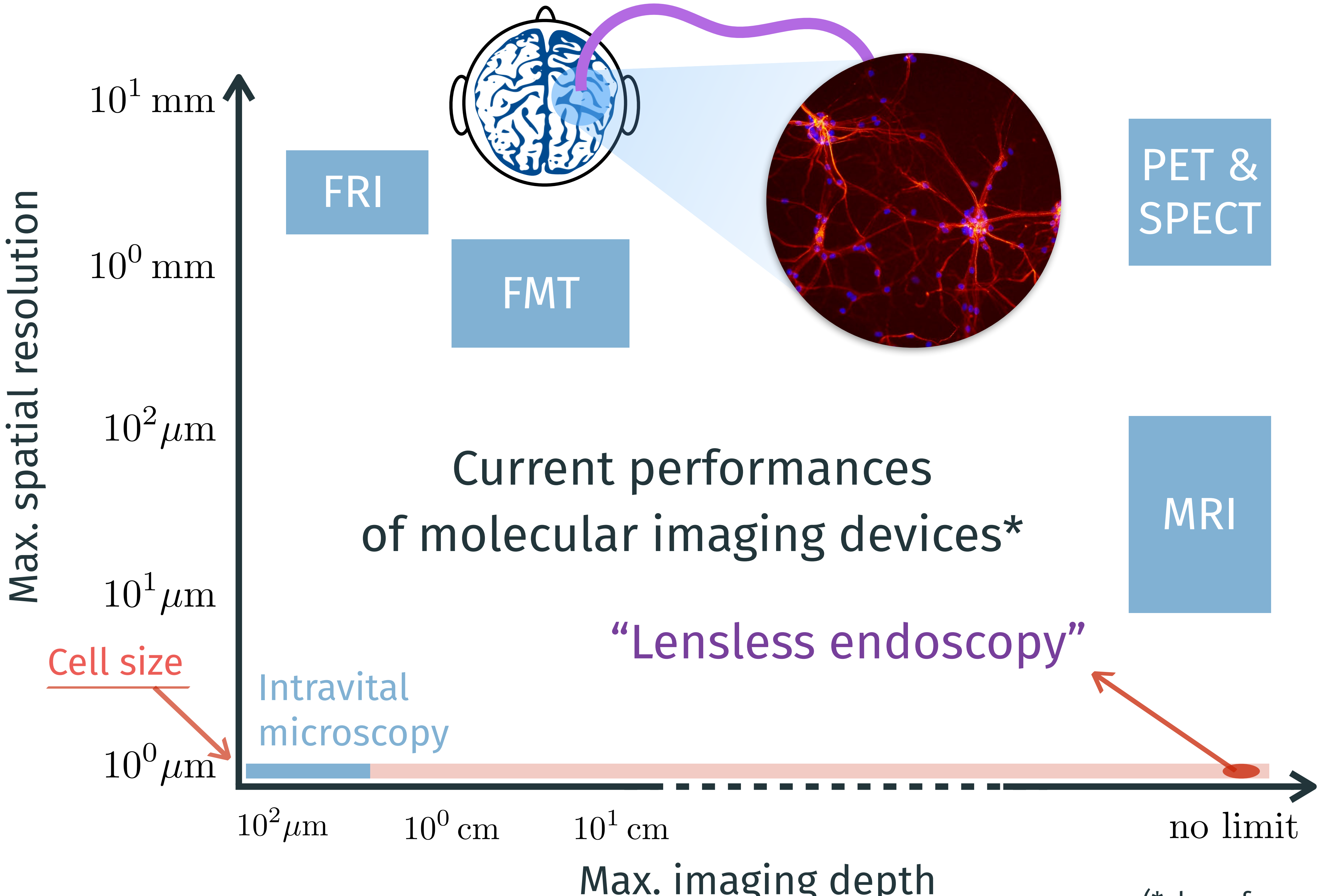
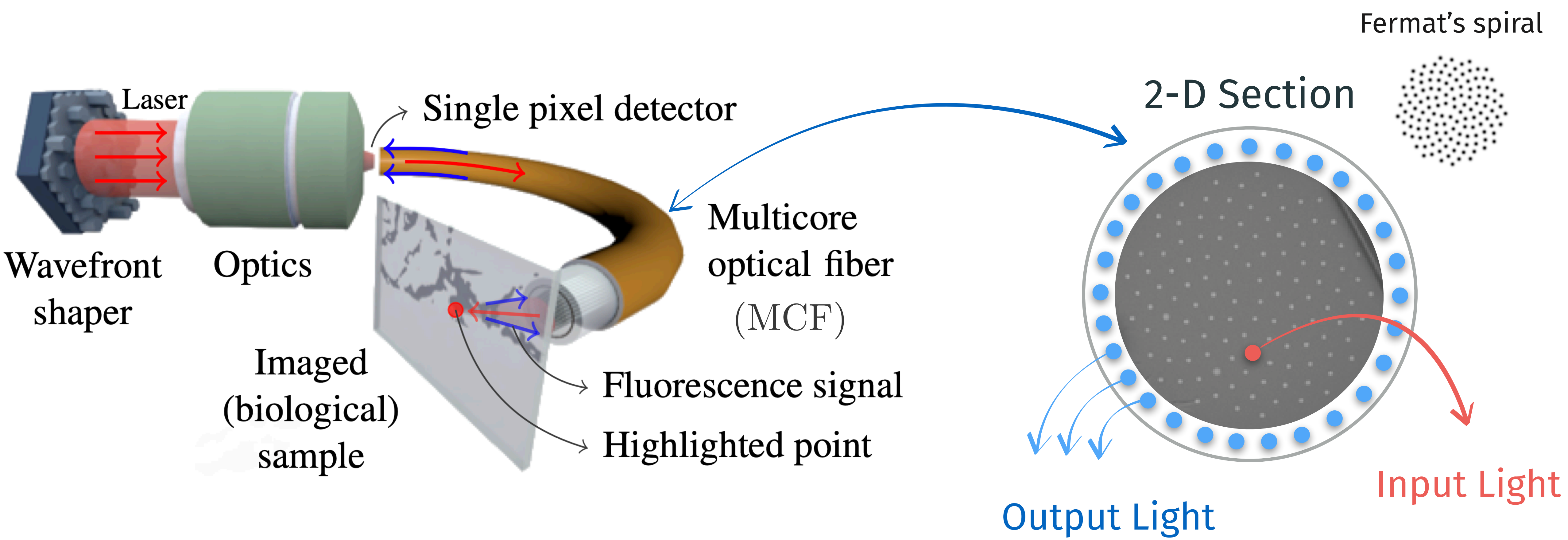


(*: here for small animals)

Rudin, M., & Weissleder, R. (2003). Molecular imaging in drug discovery and development. *Nature reviews Drug discovery*, *2*(2), 123-131.

Lensless endoscopy: focused mode



E. R. Andresen, S. Sivankutty, V. Tsvirkun, et al., "Ultrathin endoscopes based on multicore fibers and adaptive optics: status and perspectives," Journal of Biomedical Optics, 2016.

S. Sivankutty, V. Tsvirkun, O. Vanvincq, et al., "Nonlinear imaging through a fermat's golden spiral multicore fiber," Optics letters, 2018.

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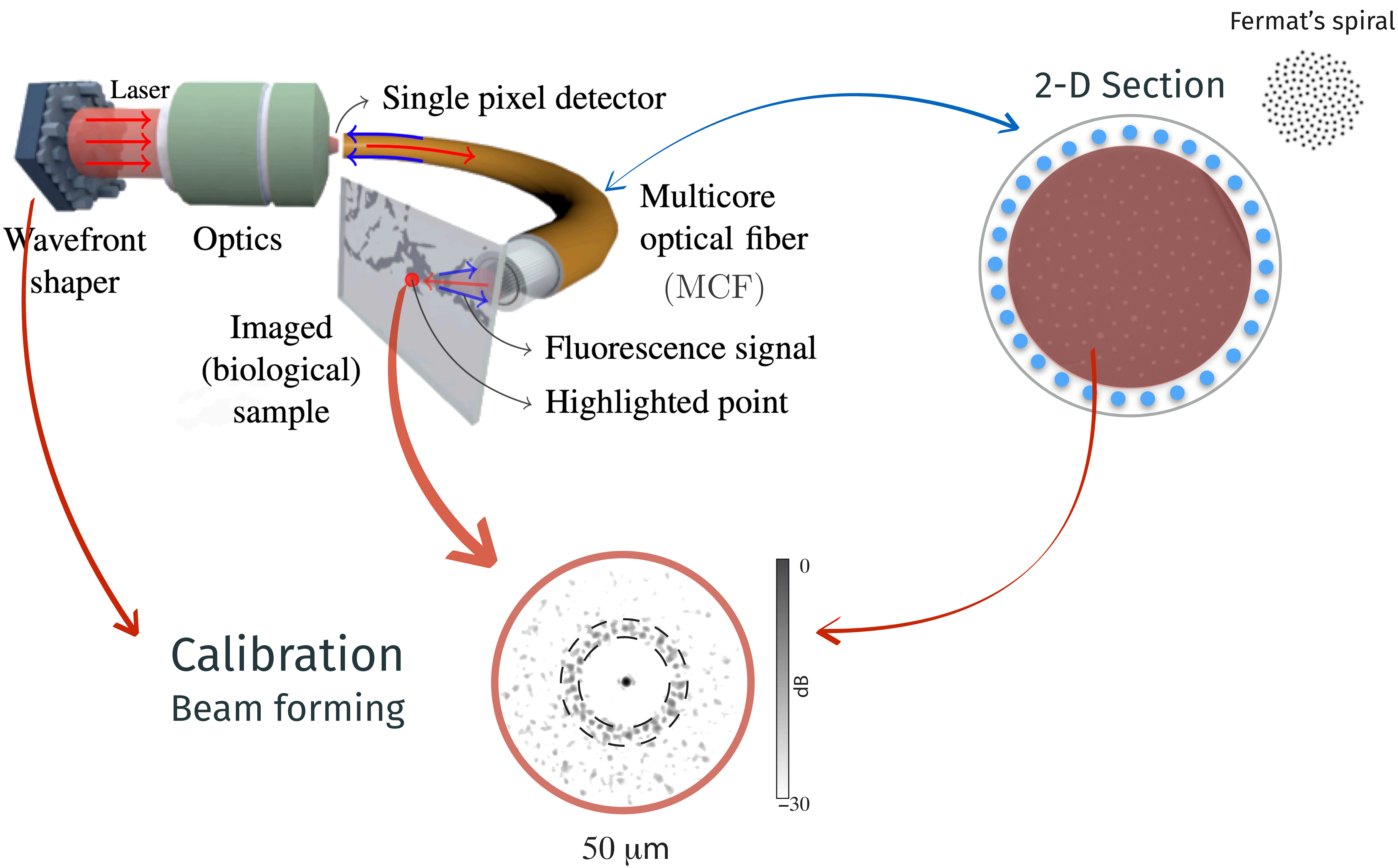

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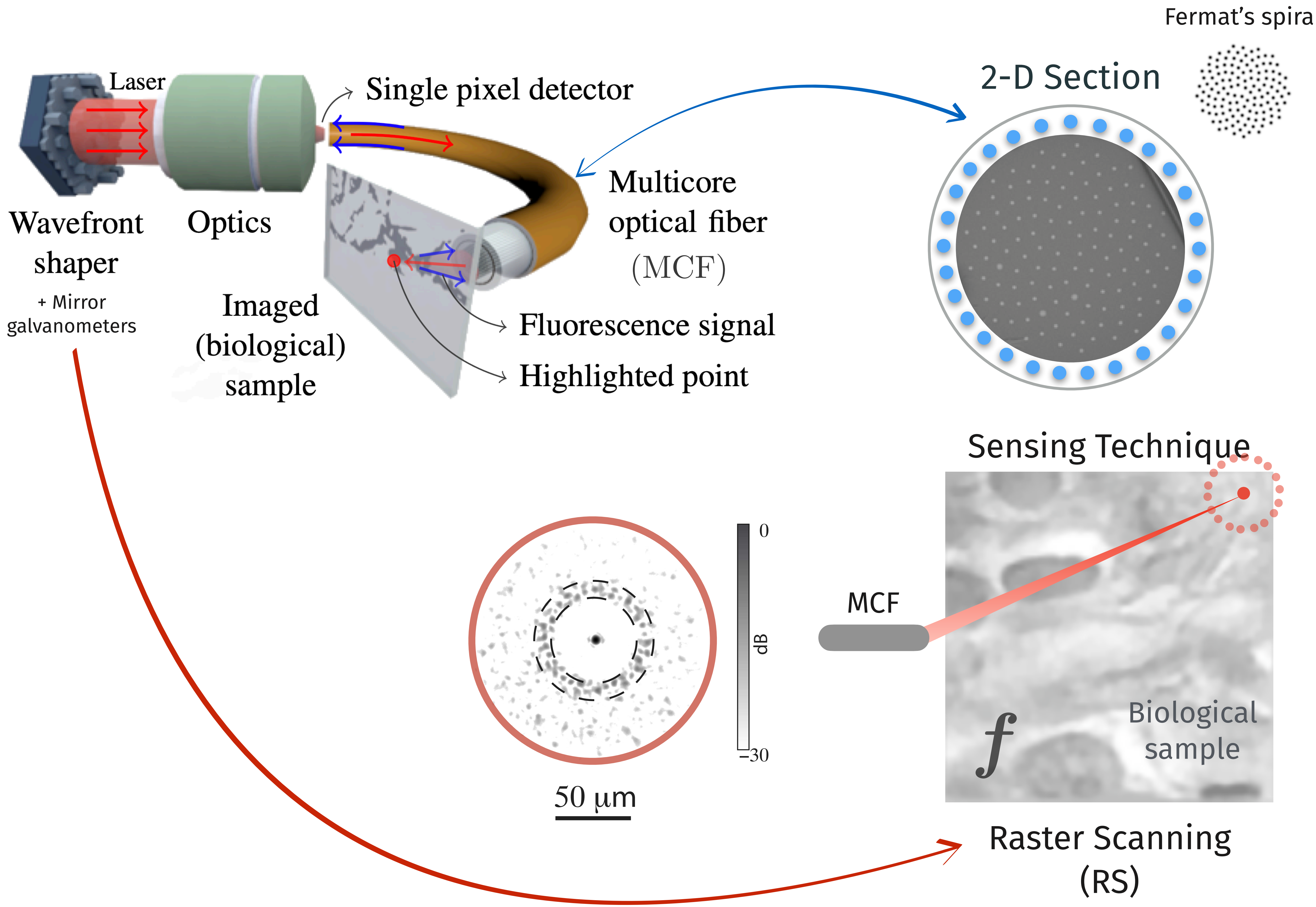

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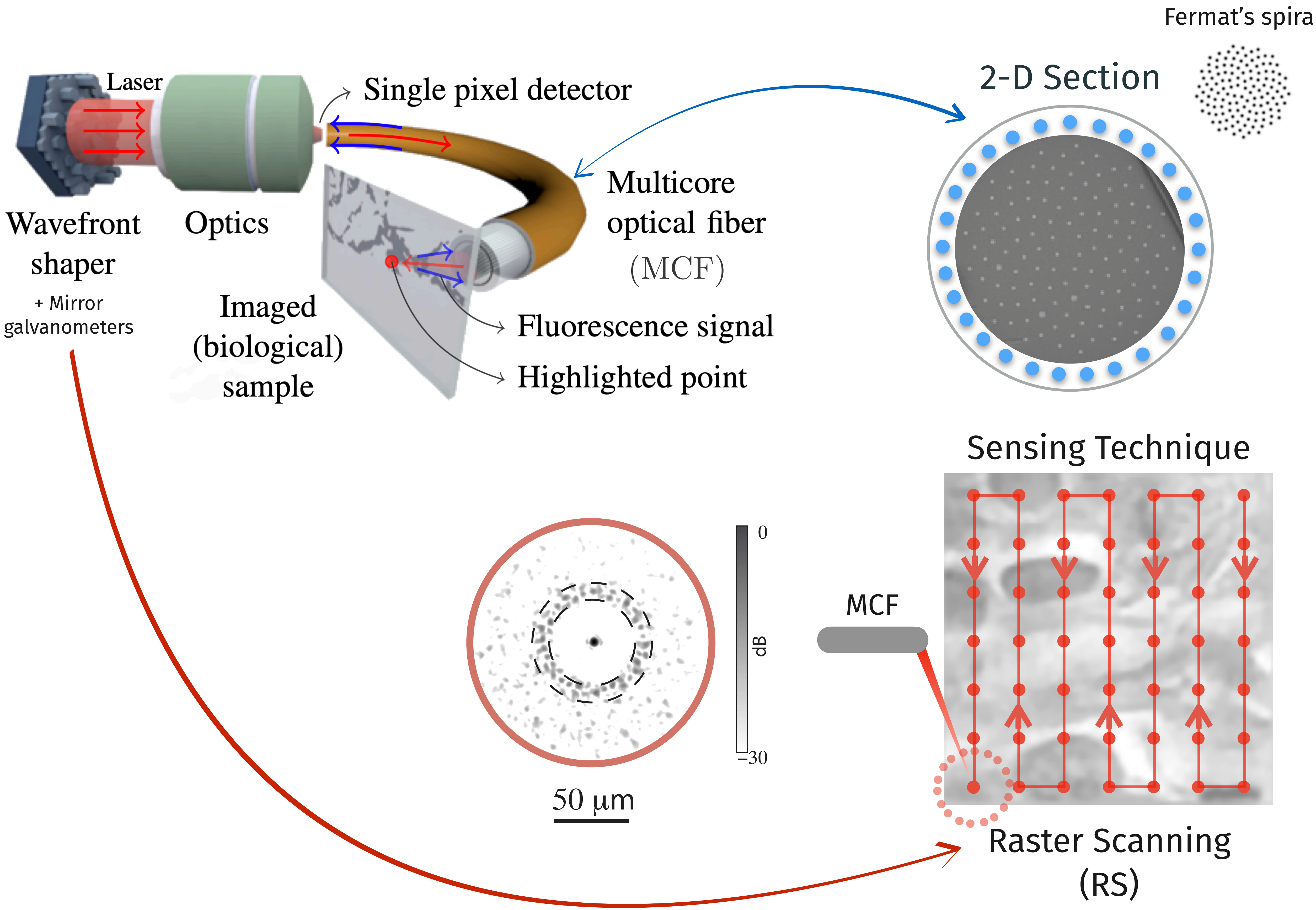

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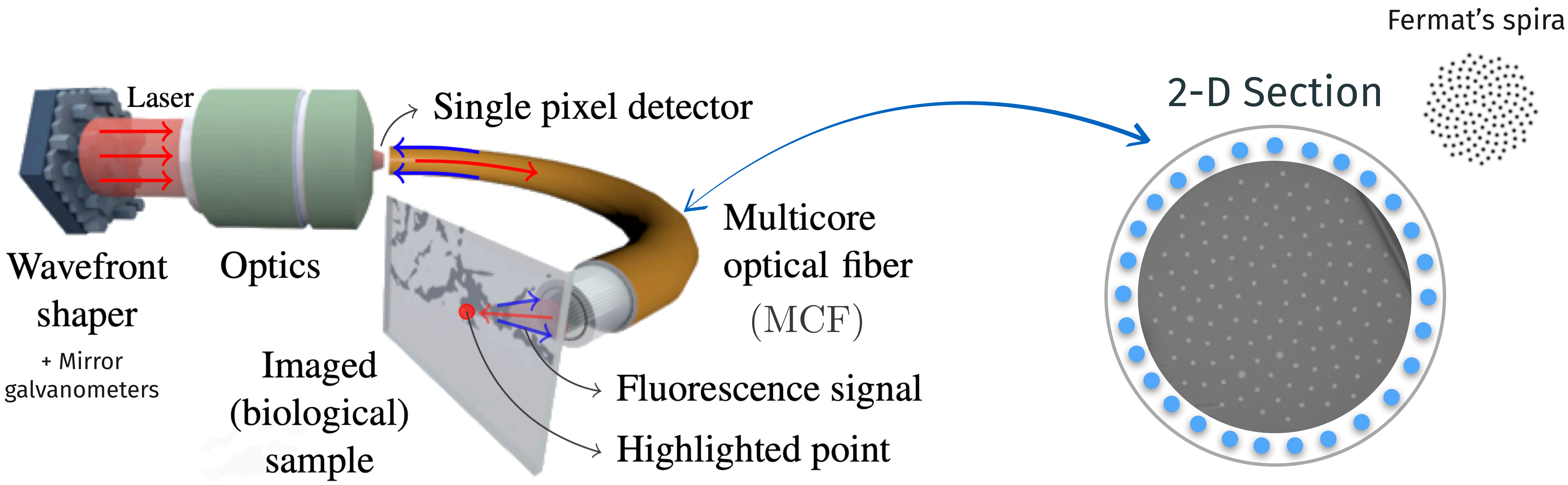
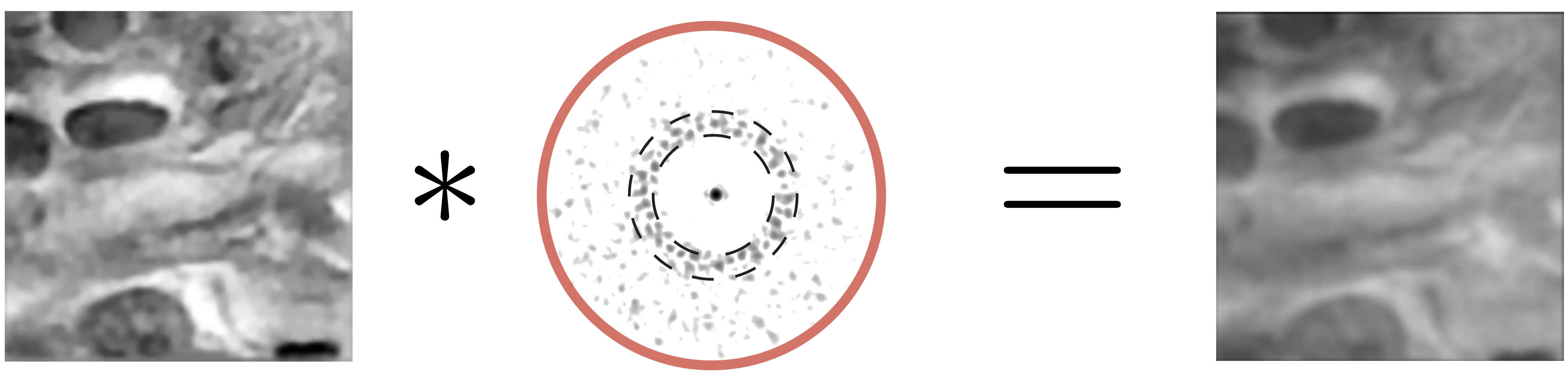
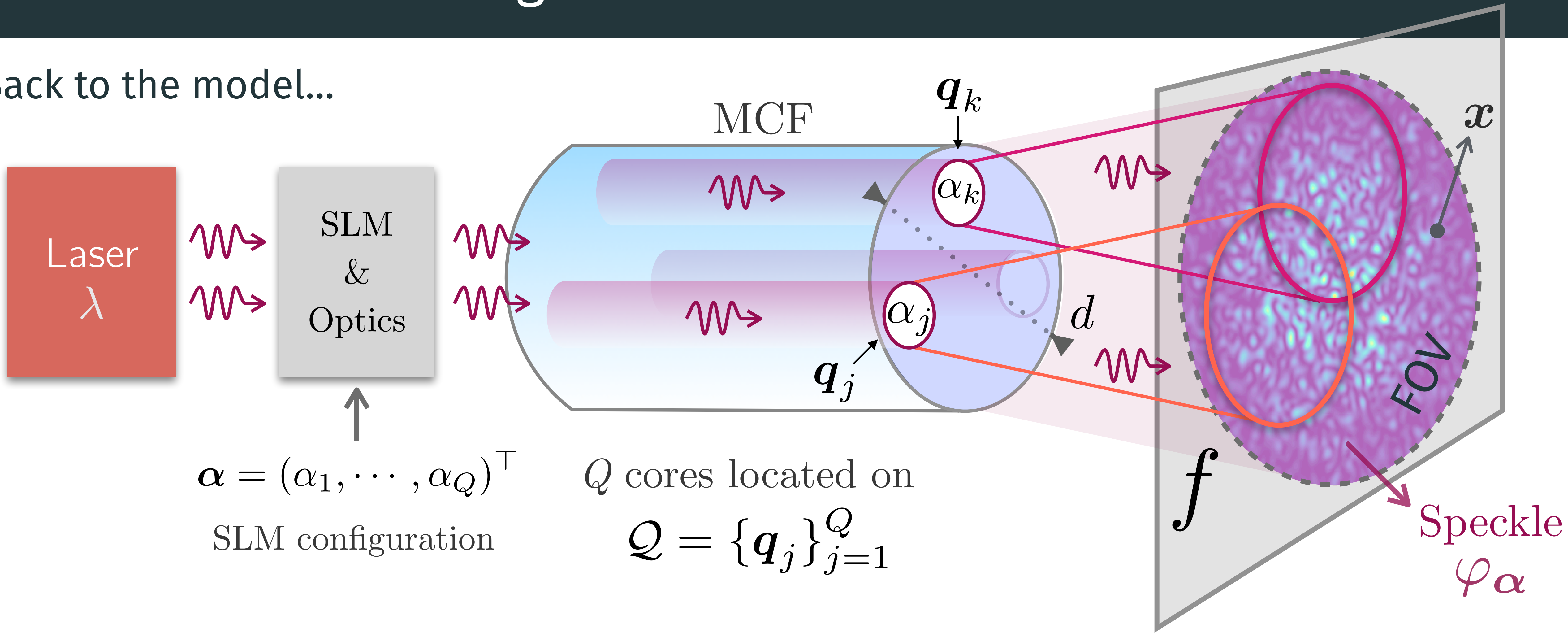




Sensing model

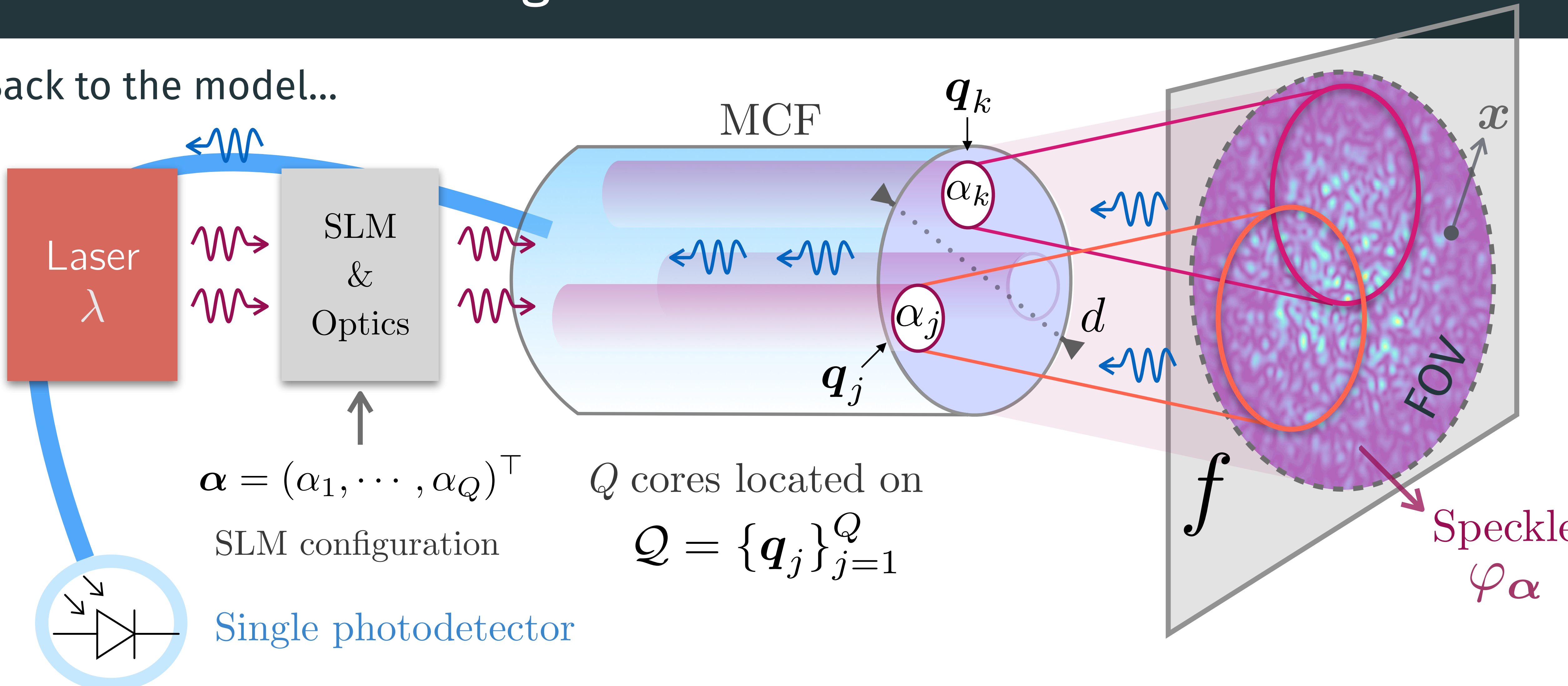
Direct Imaging

Back to the model...



A closer look to sensing model

Back to the model...

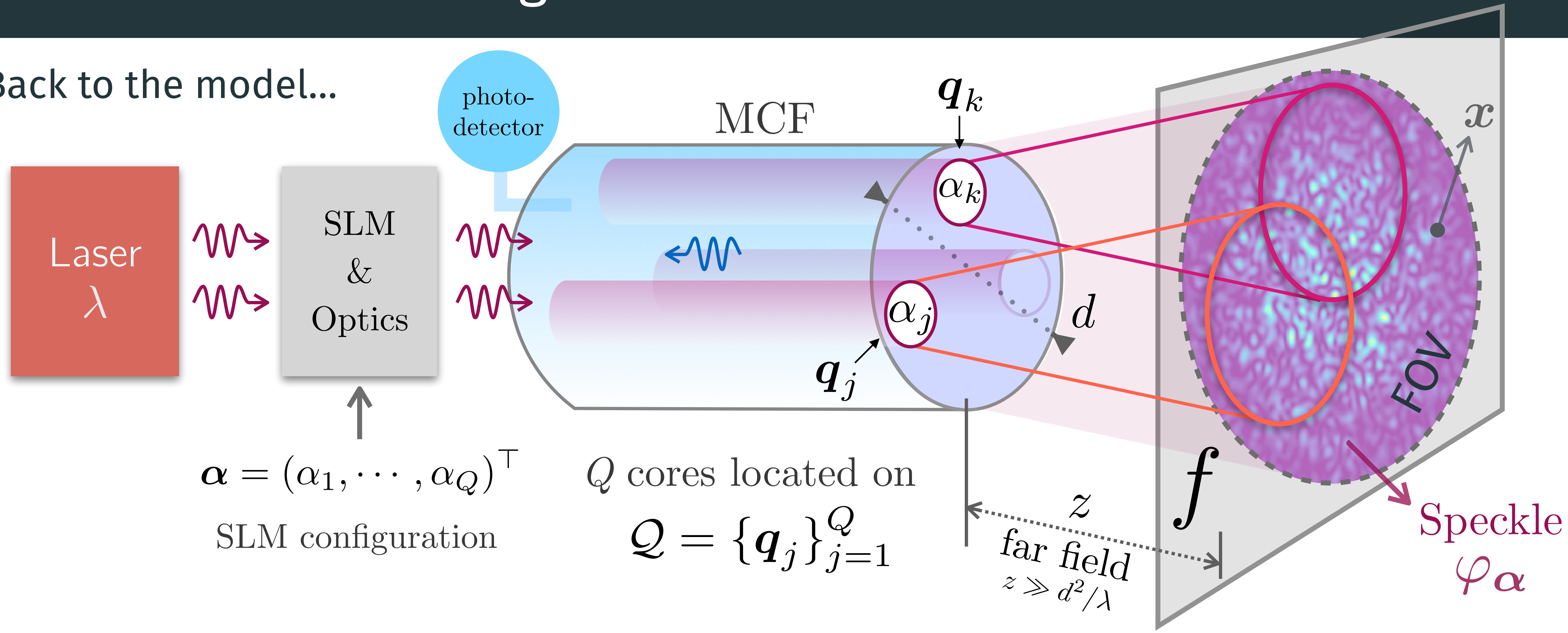


$$y_{\boldsymbol{\alpha}} \propto \int_{\mathbb{R}^2} \varphi_{\boldsymbol{\alpha}}(\boldsymbol{x}) f(\boldsymbol{x}) \, \mathrm{d}^2\boldsymbol{x} = \langle \varphi_{\boldsymbol{\alpha}}, f \rangle$$

Measurement model

A closer look to sensing model

Back to the model...



Speckles are interferences: (Under far-field approximation)

$$\varphi_{\boldsymbol{\alpha}}(\boldsymbol{x}) \propto \underbrace{w(\boldsymbol{x})}_{\text{FOV window}} \sum_{j,k=1}^{Q} \alpha_j \alpha_k^* e^{\frac{2\pi \mathrm{i}}{\lambda z}(\boldsymbol{q}_j - \boldsymbol{q}_k)^\top \boldsymbol{x}}$$

Compressive Sensing? Gaussian pattern?

(noiseless) Interferometric sensing model

Given $\varphi_{\boldsymbol{\alpha}}(\boldsymbol{x}) = w(\boldsymbol{x}) \sum_{j,k=1}^{Q} \alpha_j \alpha_k^* e^{\frac{2\pi \mathrm{i}}{\lambda z}(\boldsymbol{q}_j - \boldsymbol{q}_k)^\top \boldsymbol{x}}$, we get

$$\langle \varphi_{\boldsymbol{\alpha}}, f \rangle = \boxed{\sum_{j,k=1}^{Q} \alpha_j \alpha_k^* \Big[\int_{\mathbb{R}^2} e^{\frac{2\pi \mathrm{i}}{\lambda z}(\boldsymbol{q}_j - \boldsymbol{q}_k)^\top \boldsymbol{x}} w(\boldsymbol{x}) f(\boldsymbol{x}) \mathrm{d}\boldsymbol{x} \Big]}$$

$$\dashrightarrow \boldsymbol{\alpha}^* \boldsymbol{\mathcal{I}}[wf]\, \boldsymbol{\alpha}$$

with the (Hermitian) *interferometric matrix* $\boldsymbol{\mathcal{I}}[wf] \in \mathbb{C}^{Q \times Q}$ s.t.

$$(\boldsymbol{\mathcal{I}}[wf])_{j,k} := \int_{\mathbb{R}^2} e^{\frac{2\pi \mathrm{i}}{\lambda z}(\boldsymbol{q}_j - \boldsymbol{q}_k)^\top \boldsymbol{x}}\, w(\boldsymbol{x}) f(\boldsymbol{x})\, \mathrm{d}\boldsymbol{x}.$$

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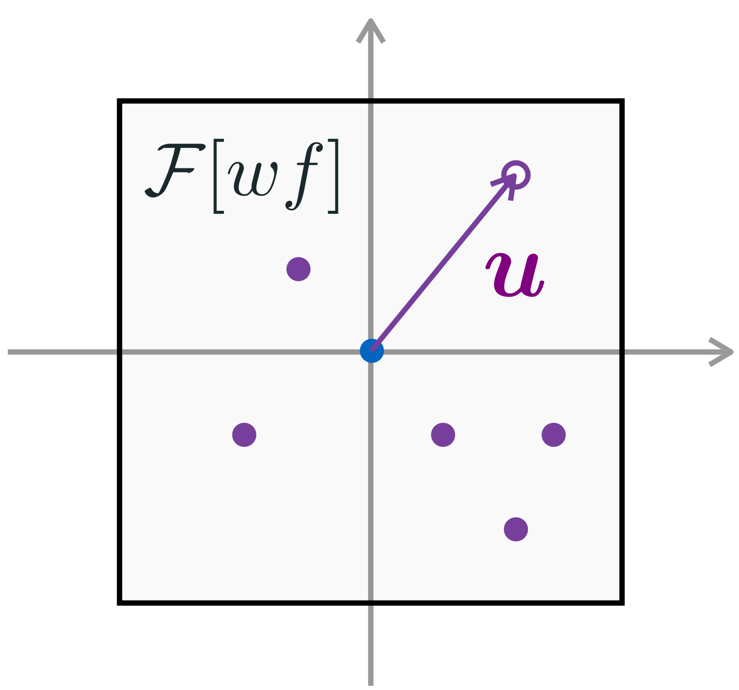

$$\boldsymbol{u} \in \mathcal{V} := \tfrac{1}{\lambda z}(\mathcal{Q} - \mathcal{Q})$$

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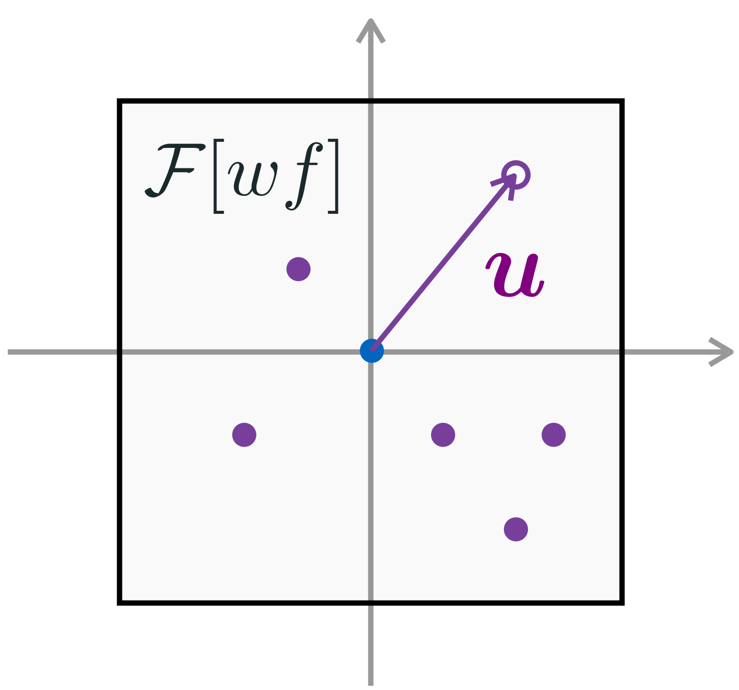
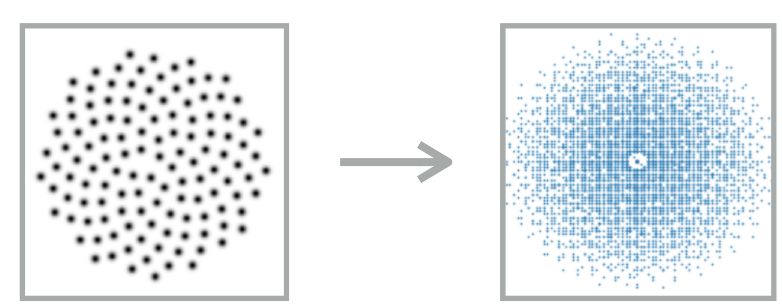


$\boldsymbol{u} \in \mathcal{V} := \frac{1}{\lambda z}(\mathcal{Q} - \mathcal{Q})$

Observation 1: denser Fourier sampling if

$$|\mathcal{V}| \simeq Q^2$$

- Lattices are bad core arrangements
- Fermat's spiral is not bad

$\mathcal{Q}$  $\mathcal{V}$

(noiseless) Interferometric sensing model

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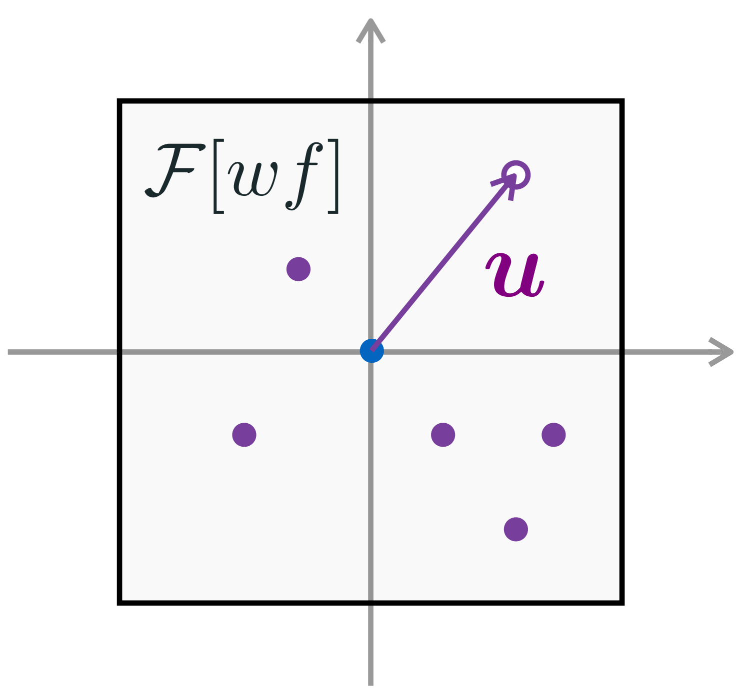

$$\boldsymbol{u} \in \mathcal{V} := \frac{1}{\lambda z}(\mathcal{Q} - \mathcal{Q})$$

Observation 2:

Low-complexity on f

→

Low-complexity on $\boldsymbol{\mathcal{I}}$.

e.g., sparsity → low-rank

(noiseless) Interferometric sensing model

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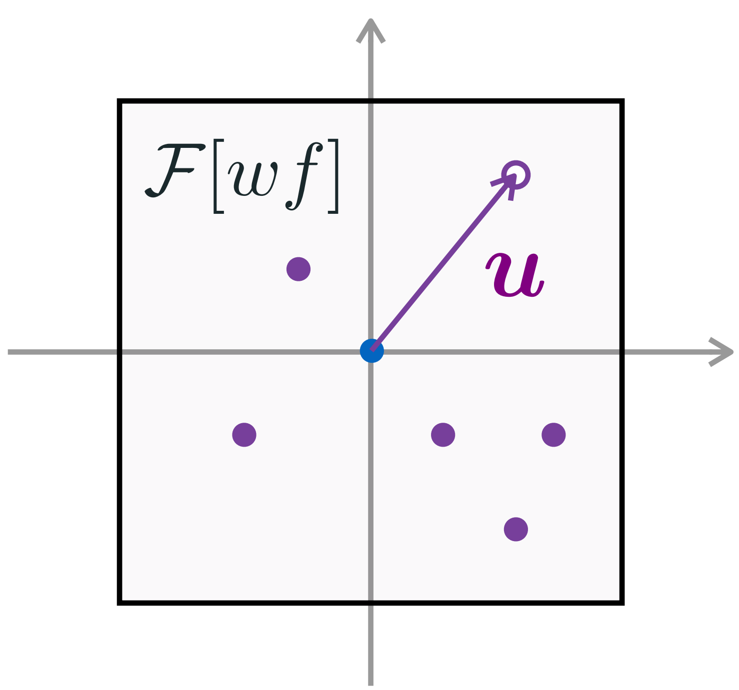

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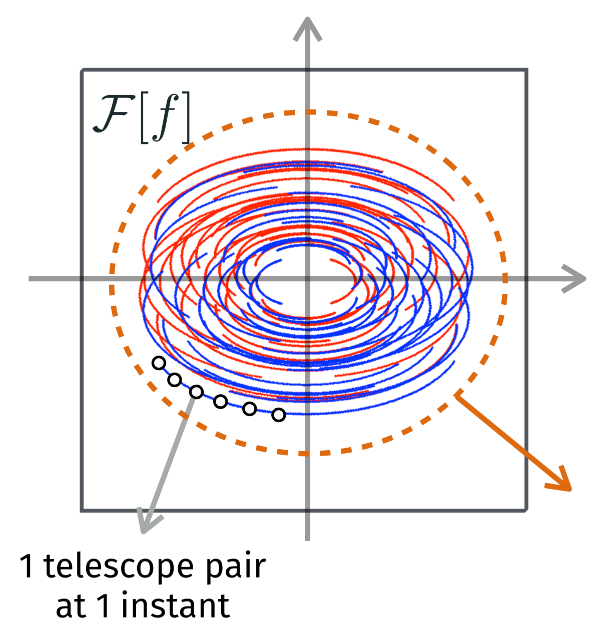
Observation 3: Similarity with radioastronomy!



visibilities $\mathcal{V}$

Noisy model:

$$y_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}^* \boldsymbol{\mathcal{I}}[wf]\, \boldsymbol{\alpha} + \text{noise} = \underbrace{\langle \boldsymbol{\alpha}\, \boldsymbol{\alpha}^*, \boldsymbol{\mathcal{I}}[wf] \rangle_{\mathrm{F}}}_{\substack{\text{Rank-one projection} \\ \text{of } \boldsymbol{\mathcal{I}}[wf]}} + \text{noise},$$

(or ROP)

General model: Over m SLMs configs $\{\boldsymbol{\alpha}_j\}_{j=1}^m$, we get

$$\boldsymbol{y} = (y_{\boldsymbol{\alpha}_1}, \cdots, y_{\boldsymbol{\alpha}_m})^\top = \mathcal{A}(\boldsymbol{\mathcal{I}}[wf]) + \text{noise},$$

with the ROP operator: $\mathcal{A}(\boldsymbol{M}) := \{\langle \boldsymbol{\alpha}_j \boldsymbol{\alpha}_j^*, \boldsymbol{M} \rangle_{\mathrm{F}}\}_{j=1}^m$.

Chen, Y., Chi, Y., & Goldsmith, A. J. (2015). Exact and stable covariance estimation from quadratic sampling via convex programming. *IEEE Transactions on Information Theory*, *61*(7), 4034-4059.

Cai, T. T., & Zhang, A. (2015). ROP: Matrix recovery via rank-one projections. The Annals of Statistics, 43(1), 102-138.

Composition of two sensing methods

$$\boldsymbol{y} = (y_{\boldsymbol{\alpha}_1}, \cdots, y_{\boldsymbol{\alpha}_m})^\top = \underbrace{\mathcal{A}}_{\textcircled{2}\; m \times Q^2}(\overbrace{\mathcal{I}[wf]}^{Q \times Q\; \textcircled{1}}) + \text{noise},$$

Sample complexities of interest:

② Does $\mathcal{A}$ capture enough from $\mathcal{I}$? $\leftrightarrow$ m big enough?

① Does $\mathcal{I}$ capture enough from f? $\leftrightarrow$ Q big enough?

Core arrangement?

Composition of two sensing methods

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Sample complexities of interest:

2. Does $\mathcal{A}$ capture enough from $\mathcal{I}$? $\leftrightarrow$ m big enough?
1. Does $\mathcal{I}$ capture enough from f? $\leftrightarrow$ Q big enough?
   Core arrangement?

A few answers from a few simplifications ...

Theory + Simulations + Experimental results

Theoretical guarantees: Assumptions (6)

H1 **Bounded FOV** : $\quad \operatorname{supp} w \subset [-\frac{L}{2}, \frac{L}{2}] \times [-\frac{L}{2}, \frac{L}{2}]$

H2 **Bandlimited f** :

$\rightarrow$ (H1 & H2) $\boldsymbol{f} \in \mathbb{R}^N$ = sampling of $w(x)f(x)$ over a N pixel grid.

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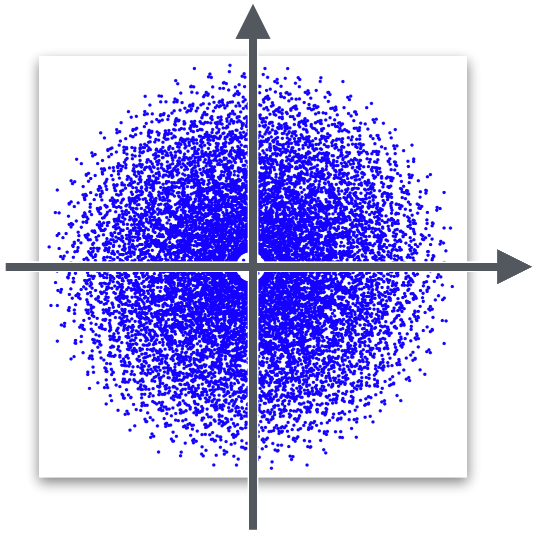

H3 K-sparse $\boldsymbol{f} \in \mathbb{R}^N$ (canonical basis)

H4 Distinct, on-grid, non-zero visibilities

$$|\mathcal{V}_0| \simeq Q^2, \text{ with } \mathcal{V}_0 = \mathcal{V} \setminus \{\mathbf{0}\}$$

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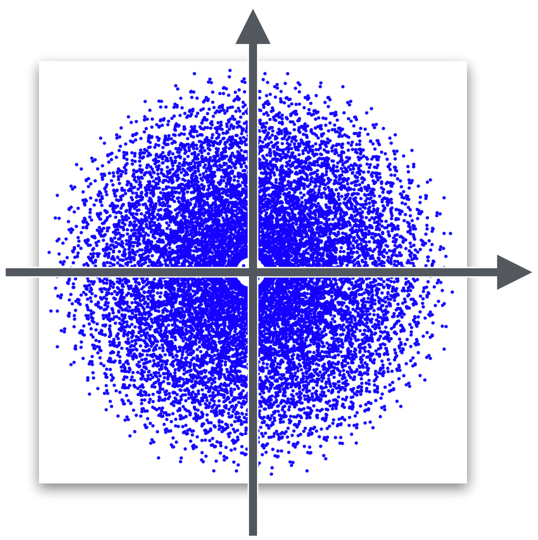

H4 Distinct, on-grid, non-zero visibilities

$$|\mathcal{V}_0| \simeq Q^2, \text{ with } \mathcal{V}_0 = \mathcal{V} \setminus \{\mathbf{0}\}$$

H5 RIP Fourier Sensing: For $\boldsymbol{\Phi} \equiv$ partial random Fourier sampling on $\mathcal{V}_0$,

$$\boldsymbol{\Phi} \text{ is } \mathrm{RIP}(K, \delta) \text{ if } |\mathcal{V}_0| \gtrsim \delta^{-2} K \mathrm{plog}(N, K, \delta),$$

$$i.e.\ \|\boldsymbol{\Phi u}\|^2 \simeq_\delta \|\boldsymbol{u}\|^2, \forall\ K\text{-sparse } \boldsymbol{u} \in \mathbb{R}^N$$

H6 Unit module *sketching vector*: $\boldsymbol{\alpha}_j \sim_{\mathrm{iid}} \boldsymbol{\alpha}_0$, with $|\alpha_k| = 1.$

Theoretical guarantees: Implications

Under previous assumptions:

$$\text{For } j, k \in [Q],\ j \neq k, \text{ up to a reshaping } \mathcal{R},$$

$$(\boldsymbol{\mathcal{I}}[w\boldsymbol{f}])_{jj} = (\boldsymbol{\mathcal{I}}[\boldsymbol{f}])_{jj} = \tfrac{1}{N}(\textstyle\sum_j f_j) \qquad \text{(DC part)}$$

$$(\boldsymbol{\mathcal{I}}[w\boldsymbol{f}])_{jk} = (\boldsymbol{\mathcal{I}}[\boldsymbol{f}])_{jk} = (\mathcal{R}(\boldsymbol{\Phi}\boldsymbol{f}))_{jk} \qquad \text{(AC part)}$$

Stabilisation of the ROP operator → *debiasing*

$$\boldsymbol{\mathcal{A}}^{\mathrm{c}} : \boldsymbol{\mathcal{J}} \in \mathcal{H}^Q \mapsto \big(\langle \boldsymbol{A}^{\mathrm{c}}_m, \boldsymbol{\mathcal{J}}\rangle\big)_{m=1}^{M} \in \mathbb{R}^M,$$

$$\text{with } \boldsymbol{A}^{\mathrm{c}}_m := \boldsymbol{\alpha}_m \boldsymbol{\alpha}_m^* - \tfrac{1}{M}\textstyle\sum_{j=1}^{M} \boldsymbol{\alpha}_j \boldsymbol{\alpha}_j^*$$

Equivalence: Given $\boldsymbol{y} = \boldsymbol{\mathcal{A}}(\boldsymbol{\mathcal{J}})$,

$$\underbrace{y^{\mathrm{c}}_k := y_k - \tfrac{1}{M}\textstyle\sum_{j=1}^{M} y_j}_{\text{centering}} \text{ for } k \in [M] \Rightarrow \boldsymbol{y}^{\mathrm{c}} = \boldsymbol{\mathcal{A}}^{\mathrm{c}}(\boldsymbol{\mathcal{J}}).$$

Theoretical guarantees: Proposed reconstruction

$$\tilde{\boldsymbol{f}} = \arg\min_{\boldsymbol{v}\in\mathbb{R}^N} \|\boldsymbol{v}\|_1 \text{ s.t. } \big\|\boldsymbol{y}^{\mathrm{c}} - \underbrace{\varpi\,\boldsymbol{\mathcal{A}}^{\mathrm{c}}\big(\mathcal{R}(\boldsymbol{\Phi}\boldsymbol{v})\big)}_{=:\boldsymbol{\mathcal{B}}(\boldsymbol{v})}\big\|_1 \le \epsilon$$

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$\boldsymbol{\mathcal{B}}$ has the $\mathrm{RIP}_{\ell_2/\ell_1}(K, \mathsf{m}_K, \mathsf{M}_K)$ w.h.p:

Under **H1-H6**, if $M \ge CK \ln(\frac{12eN}{K})$ and $Q(Q-1) \ge 4K\,\mathrm{plog}(N, K, \delta)$, $\exists 0 < \mathsf{m}_K < \mathsf{M}_K$ such that, w.h.p,

$$\mathsf{m}_K \|\boldsymbol{v}\| \le \tfrac{1}{M} \|\boldsymbol{\mathcal{B}}(\boldsymbol{v})\|_1 \le \mathsf{M}_K \|\boldsymbol{v}\|, \quad \forall \boldsymbol{v} \in \Sigma_K.$$

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Instance optimality:

Provided $\boldsymbol{\mathcal{B}}$ has the $\mathrm{RIP}_{\ell_2/\ell_1}(K', \mathsf{m}'_K, \mathsf{M}'_K)$, for $K' = O(K)$, $\exists C_0, D_0 > 0$,

$$\|\boldsymbol{f} - \tilde{\boldsymbol{f}}\| \leq C_0\frac{\|\boldsymbol{f}-\boldsymbol{f}_K\|_1}{\sqrt{K}} + D_0\frac{\epsilon}{M}.$$

1-D simulations: phase transition diagrams

Simplified setting:

- 1-D core arrangement, $N = 256$
- K-sparse vectors
- Random $\{\boldsymbol{\alpha}_j\}_{j=1}^{M}$
- Q, M, K varying
- 80 trials, Success if $\geq$ 40 dB

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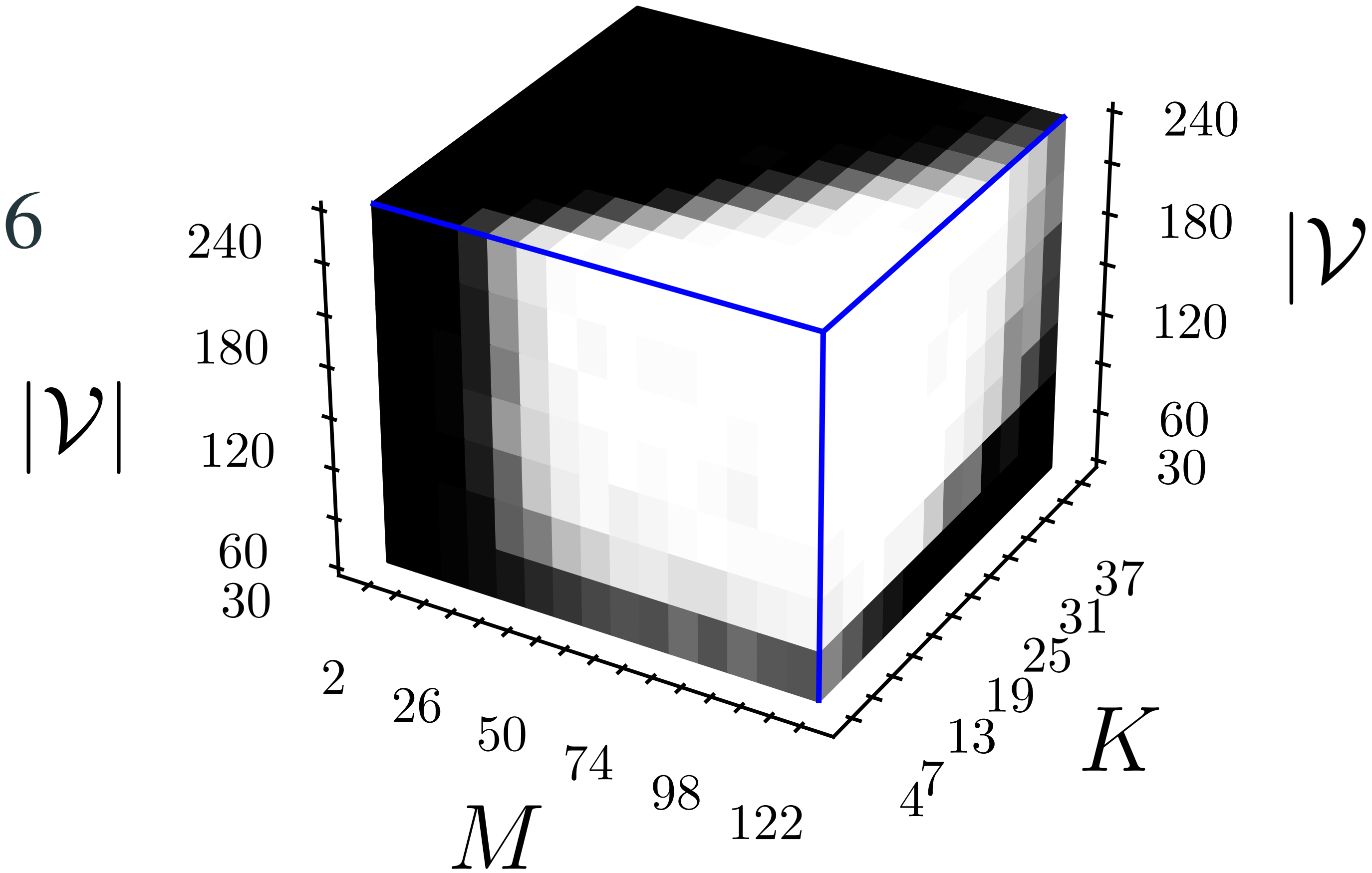
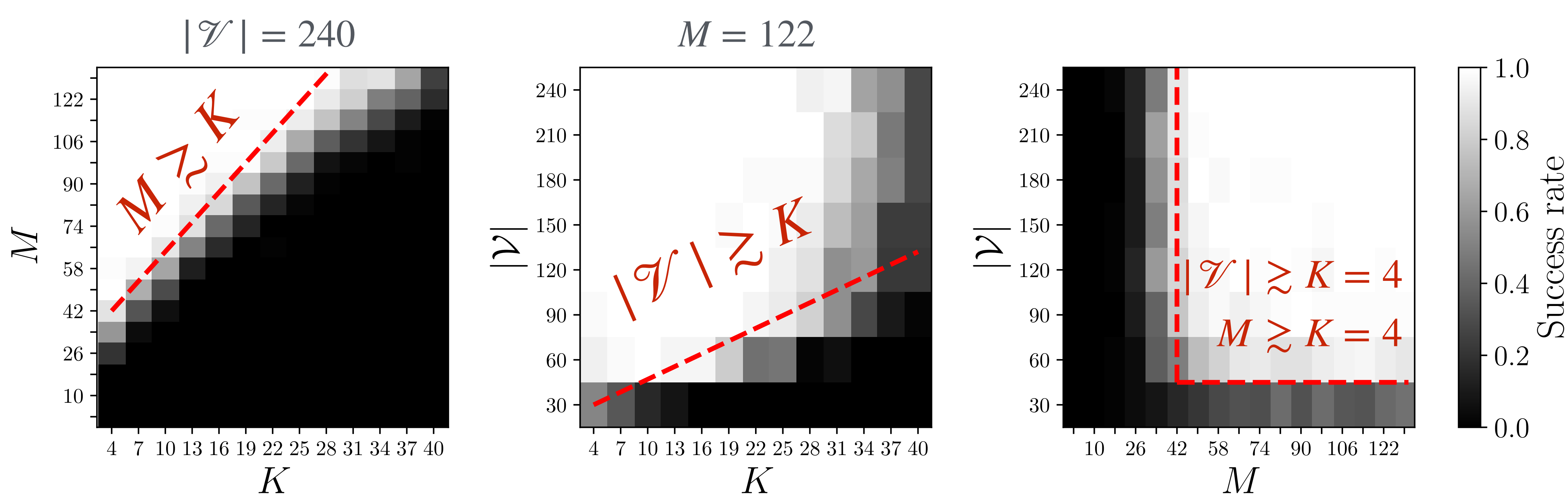
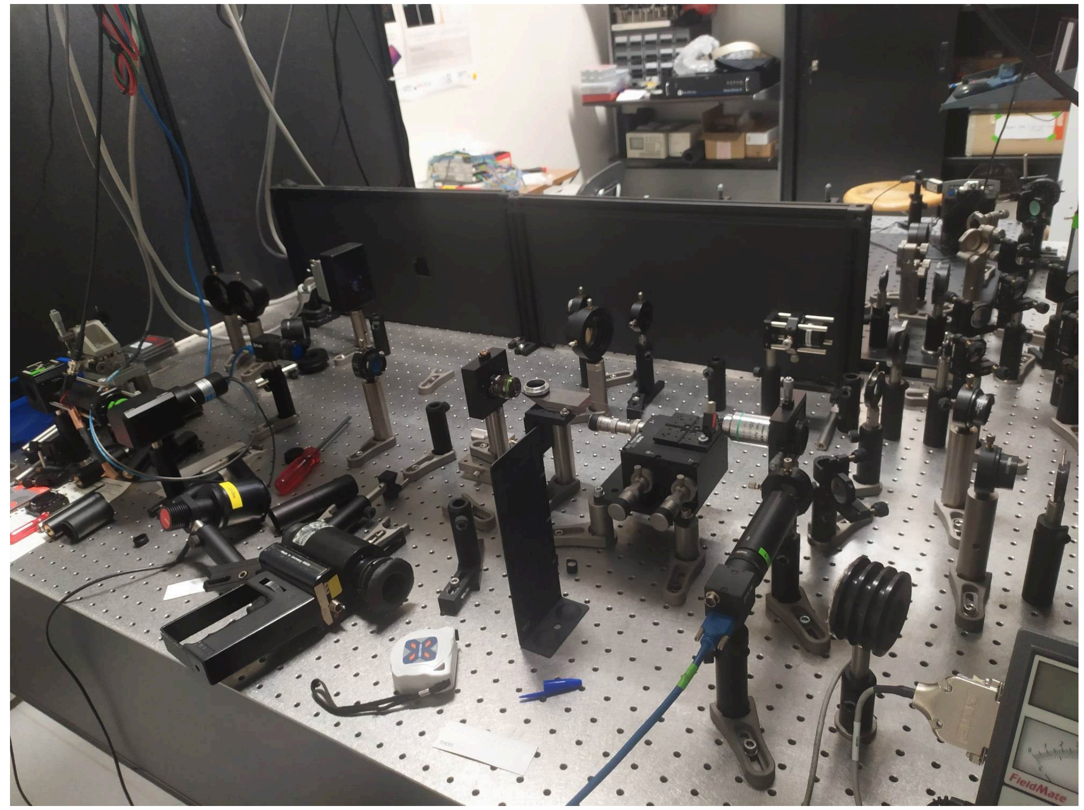
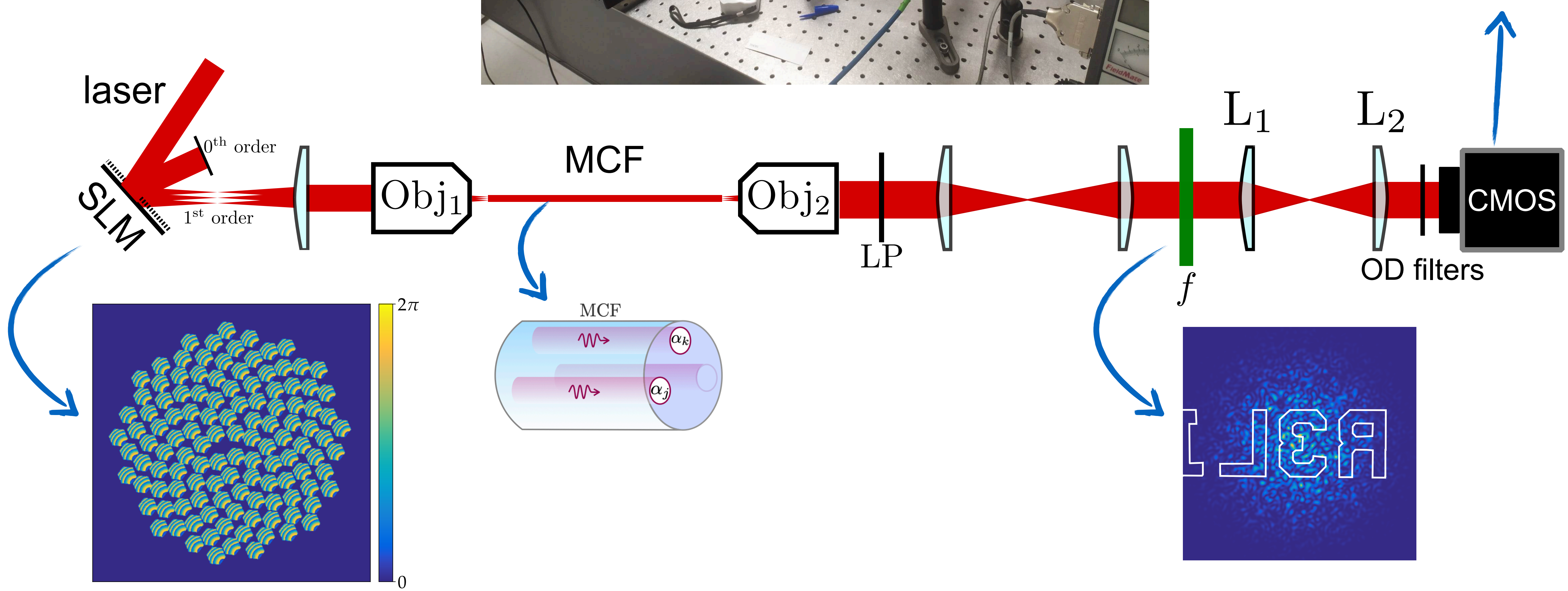




Experiments (in Institut Fresnel, France)



Single pixel
=
Pixel summation



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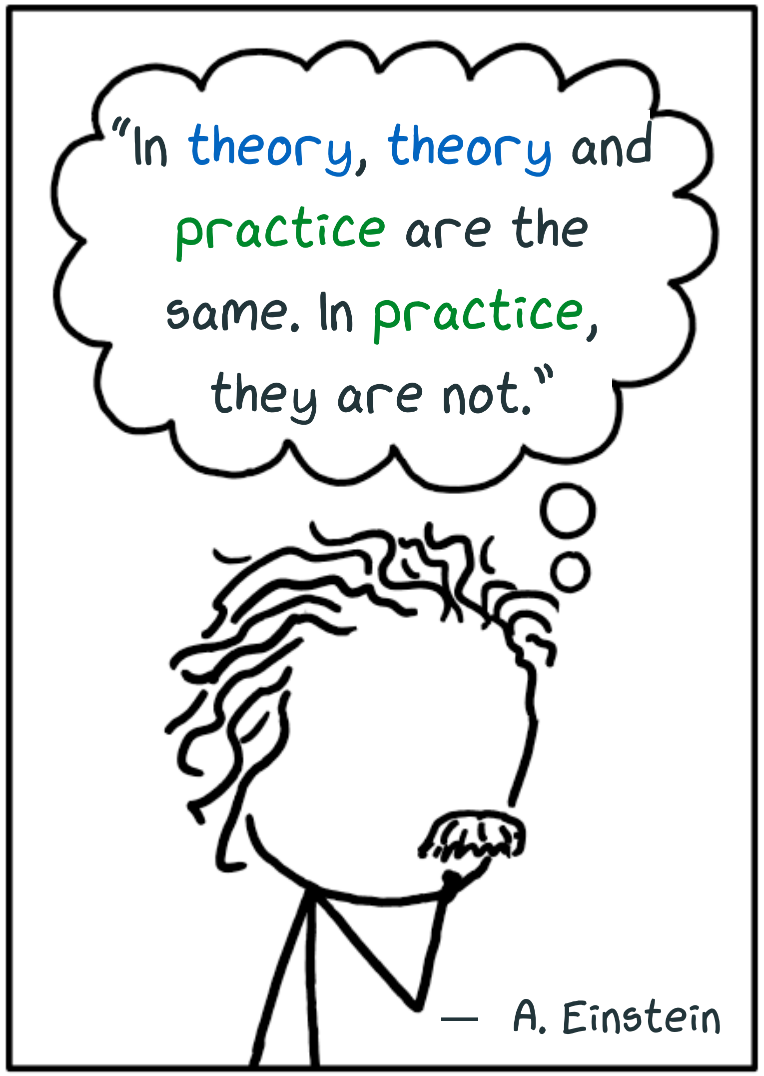

(Adapted from xkcd #1233)

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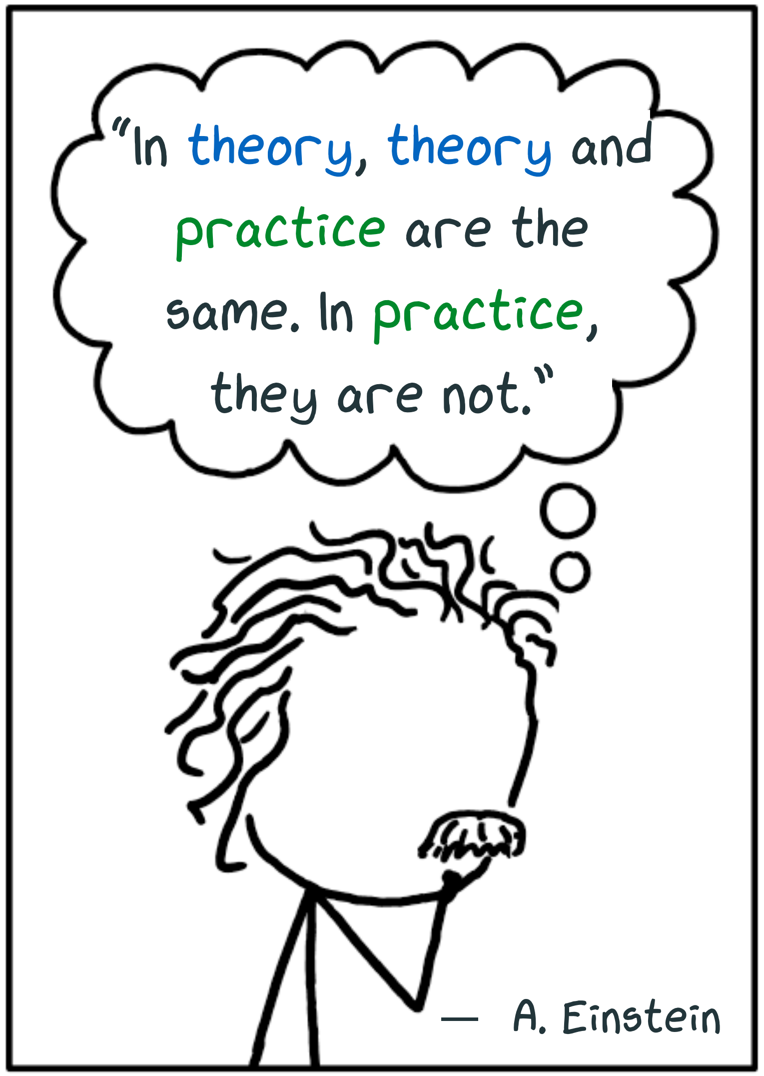


(Adapted from xkcd #1233)

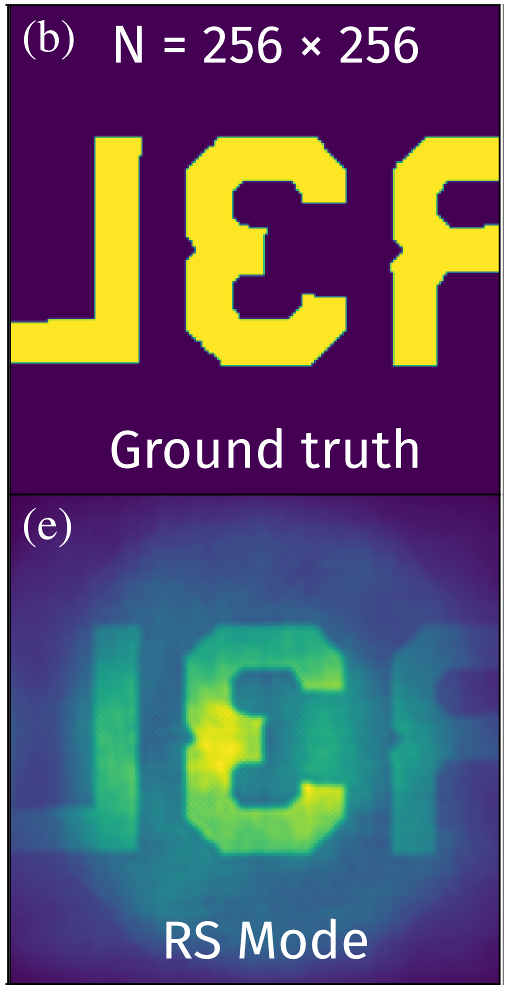
Special points of attention:

- MCF must be calibrated
- MCF system in transmission mode
- Speckle calibration (system imperfections)
  e.g., $\neq$ core radius, locations, ...

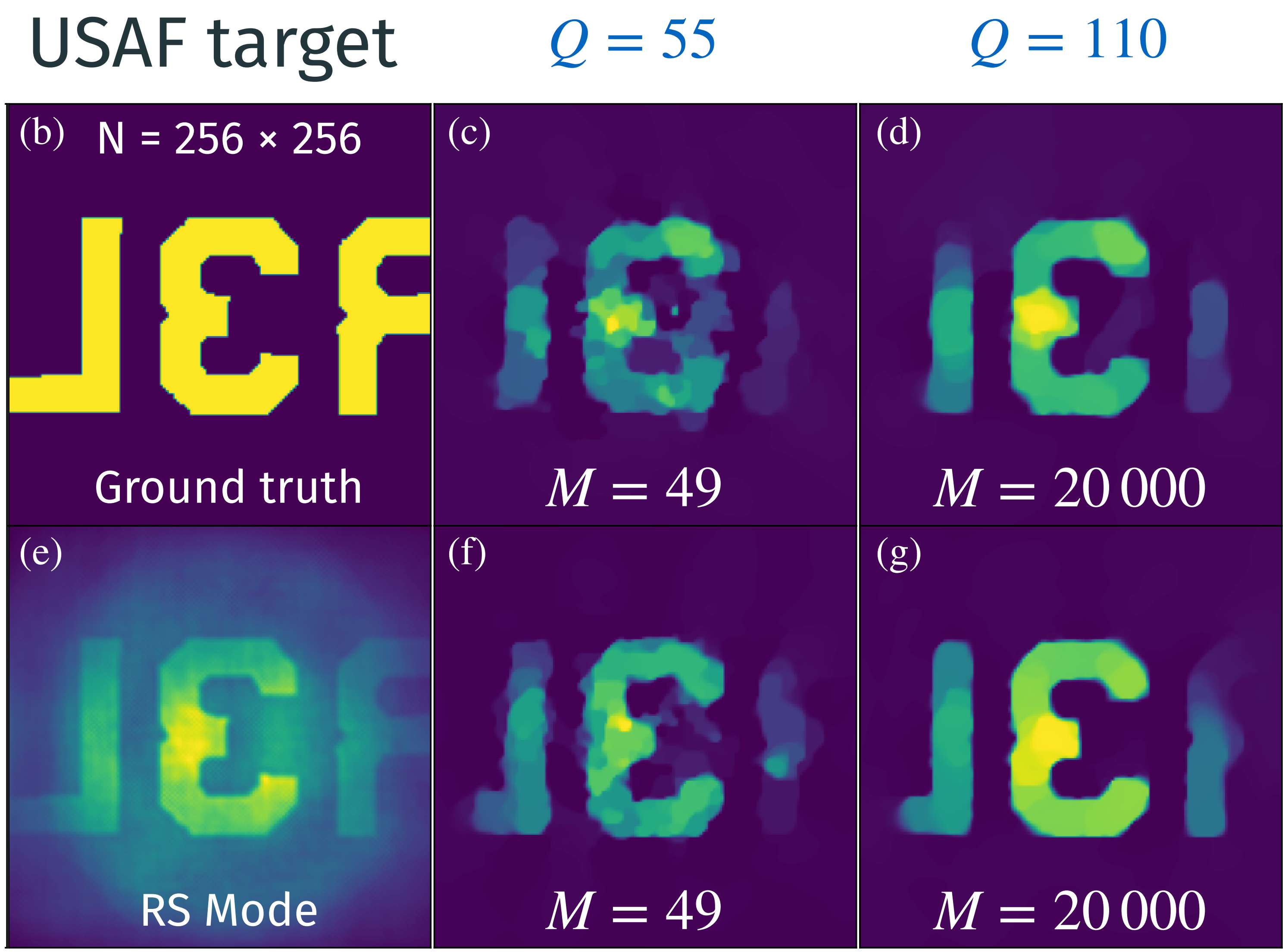
Reconstruction method: TV regulariser

$$\tilde{\boldsymbol{f}} = \arg\min_{\boldsymbol{f}} \ \tfrac{1}{2}\|\boldsymbol{y}^{\mathrm{c}} - \underbrace{\boldsymbol{\mathcal{B}}}_{\text{ROP + interfero}}(\boldsymbol{f})\|_2^2 + \underbrace{\rho}_{\text{Set empirically}}\|\boldsymbol{f}\|_{\mathrm{TV}} \ \text{s.t.}\ \boldsymbol{f} \geq 0,$$

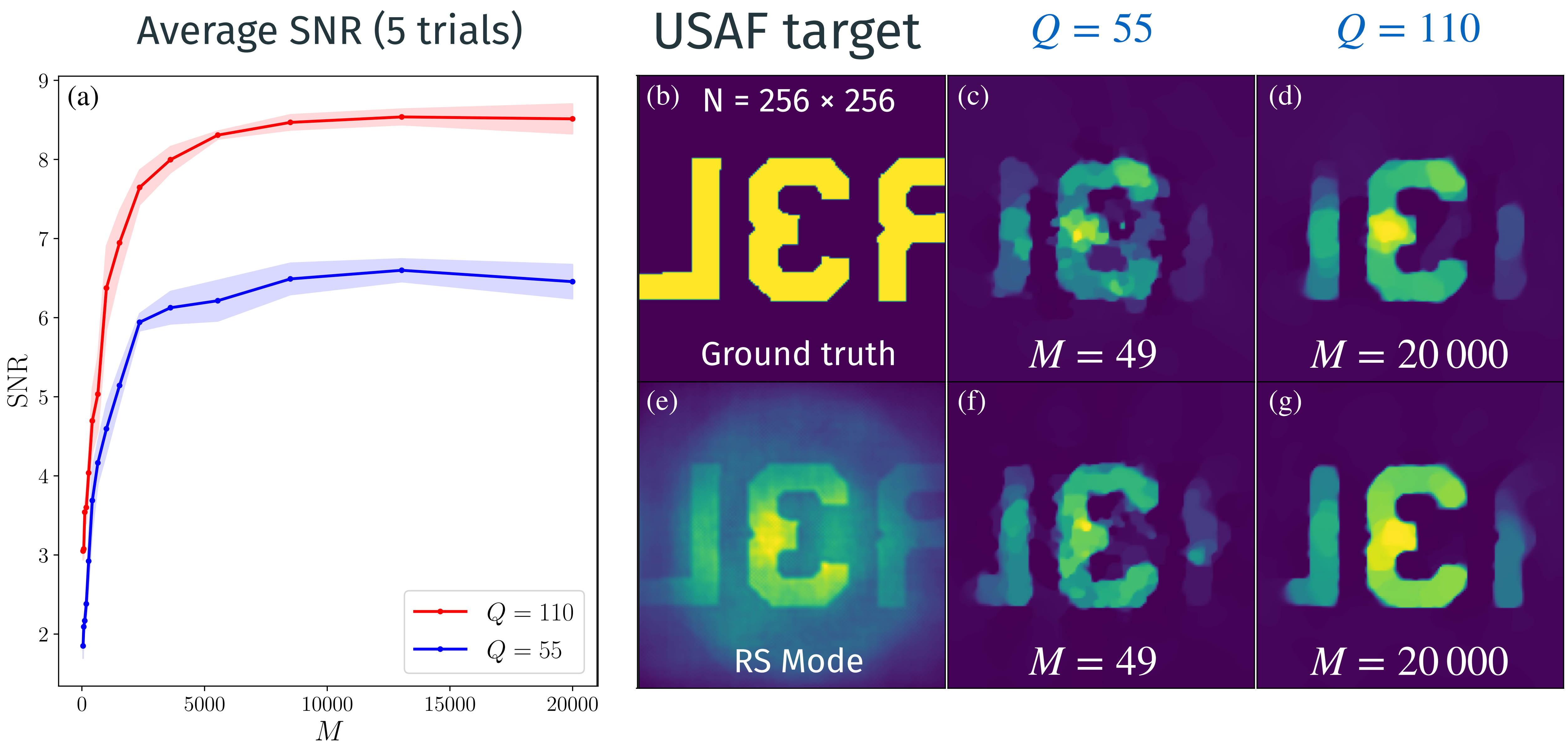
USAF target



Experiments (in Institut Fresnel, France)



Experiments (in Institut Fresnel, France)



To conclude ...

Take away messages:

- Fluorescent compressive speckle imaging (with MCF) follows an interferometric sensing model;
- This model amounts to "rank-one projecting" an interferometric matrix.
- This matrix has low-complexity

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- Fluorescent compressive speckle imaging (with MCF) follows an interferometric sensing model;
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Open questions:

- Using more advanced sparsity models:
  e.g., sparsity in levels + wavelets, weighted sparsity
- Optimization of core arrangements
- Extension to 3D imaging
- Data-driven calibration and reconstruction

Thank you for your attention!

Related bibliography:

- S. Guérit, S. Sivankutty, J. A. Lee, H. Rigneault and L. Jacques, "Compressive Imaging through Optical Fiber with Partial Speckle Scanning," under review, 2021.
- Rudin, M., & Weissleder, R. (2003). Molecular imaging in drug discovery and development. *Nature reviews Drug discovery*, *2*(2), 123-131.
- E. R. Andresen, S. Sivankutty, V. Tsvirkun, et al., "Ultrathin endoscopes based on multicore fibers and adaptive optics: status and perspectives," Journal of Biomedical Optics, 2016.
- S. Sivankutty, V. Tsvirkun, O. Vanvincq, et al., "Nonlinear imaging through a fermat's golden spiral multicore fiber," Optics letters, 2018.
- Shin, J., Bosworth, B. T., & Foster, M. A. (2017). Compressive fluorescence imaging using a multi-core fiber and spatially dependent scattering. Optics letters, 42(1), 109-112.
- Chen, Y., Chi, Y., & Goldsmith, A. J. (2015). Exact and stable covariance estimation from quadratic sampling via convex programming. *IEEE Transactions on Information Theory*, *61*(7), 4034-4059.
- Cai, T. T., & Zhang, A. (2015). ROP: Matrix recovery via rank-one projections. The Annals of Statistics, 43(1), 102-138.
- Adcock, B., Hansen, A. C., Poon, C., & Roman, B. (2017). Breaking the coherence barrier: A new theory for compressed sensing. In Forum of Mathematics, Sigma (5). Cambridge University Press.