Robust 1-Bit Compressive Sensing:

How the Sign of Random Projections Distinguishes Sparse Vectors

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Joint work with:

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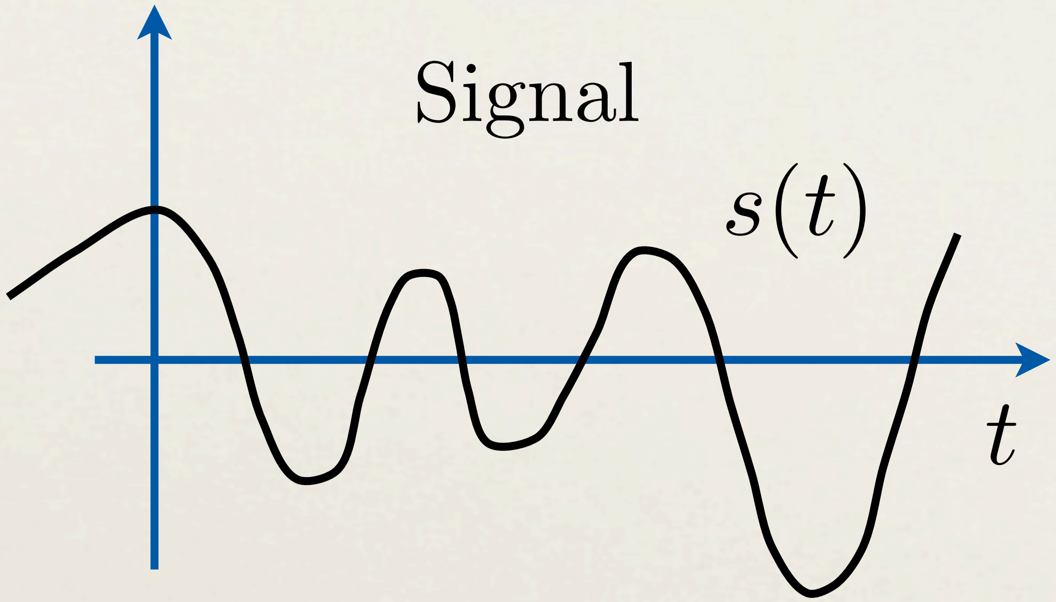
and Richard Baraniuk (Rice U., TX, USA)

FNRS Contact Group "Wavelets and applications"
Dec. 21, 2011

1-bit Sampling?



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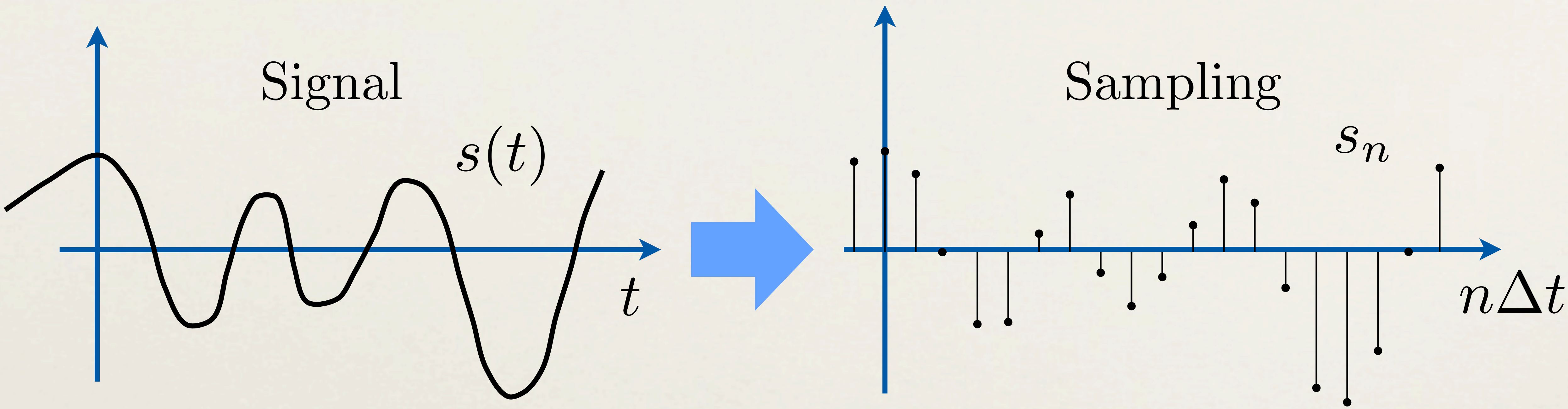

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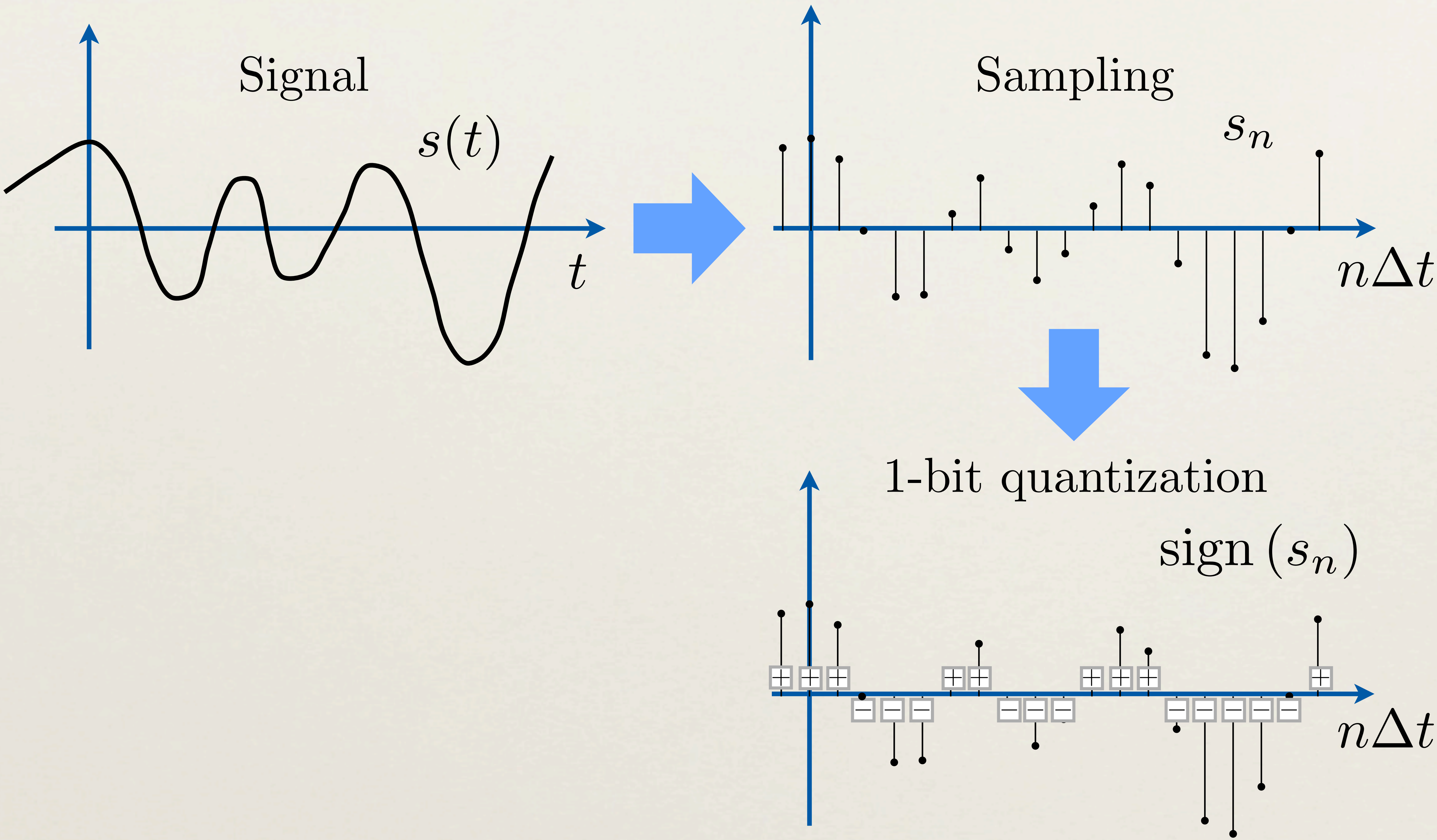

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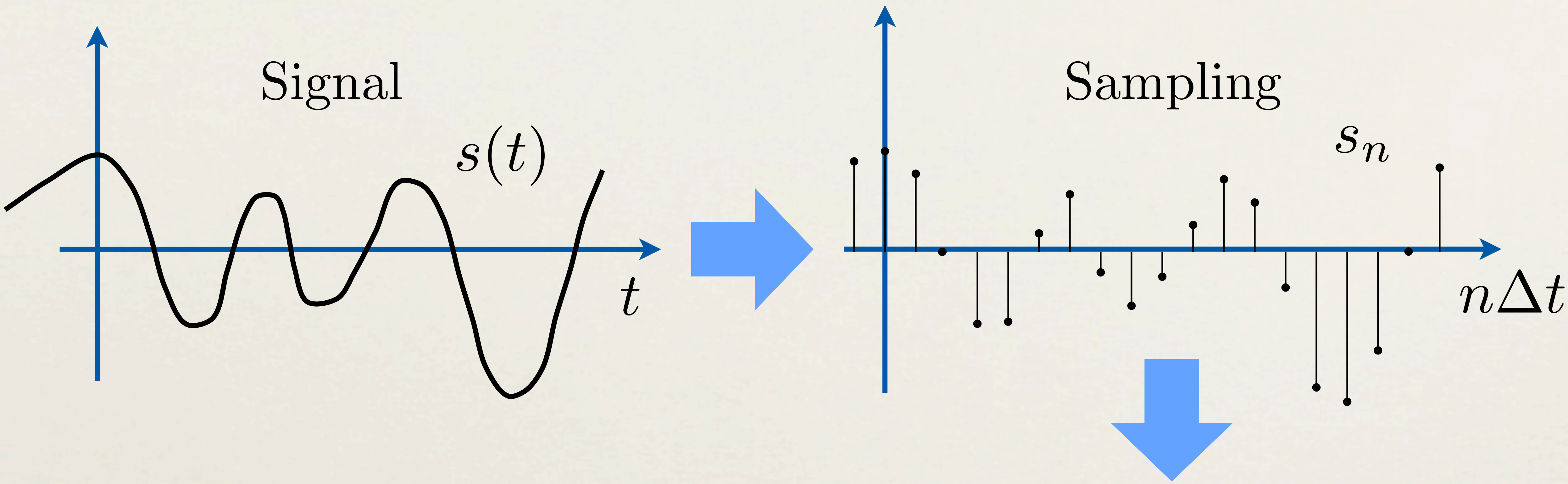
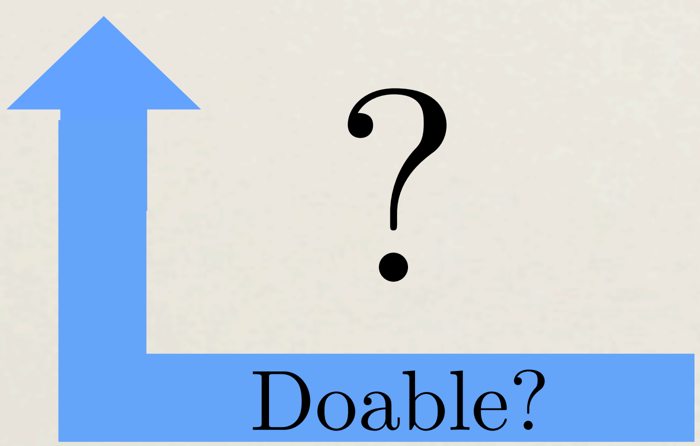
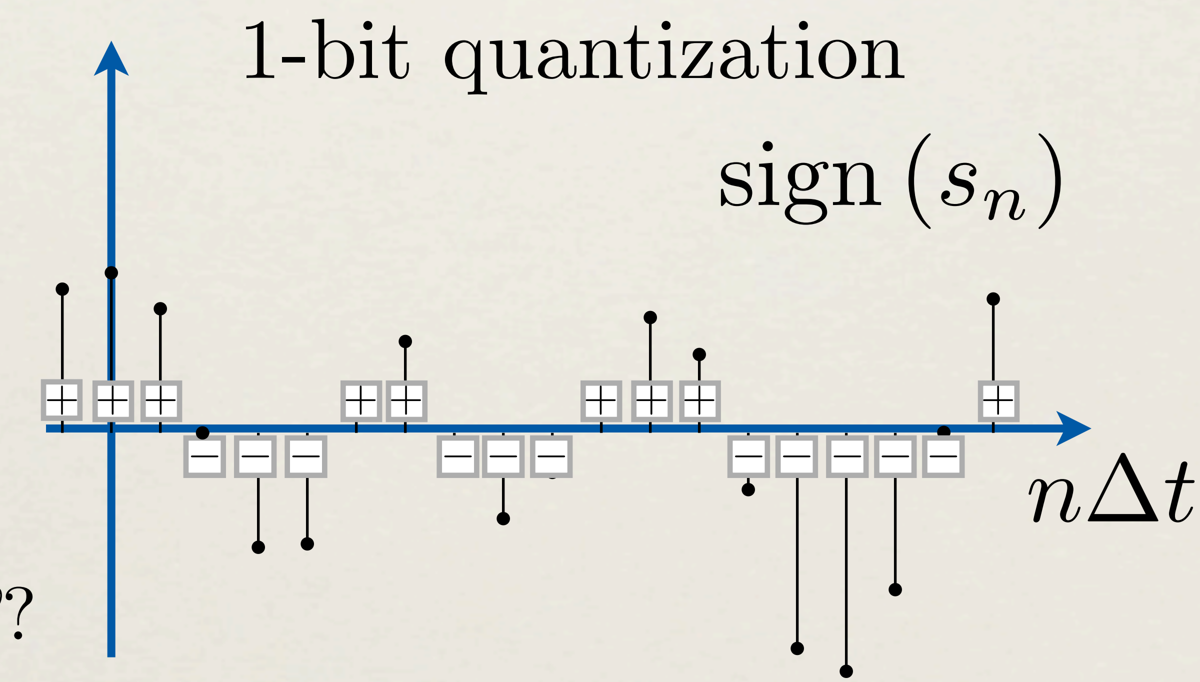
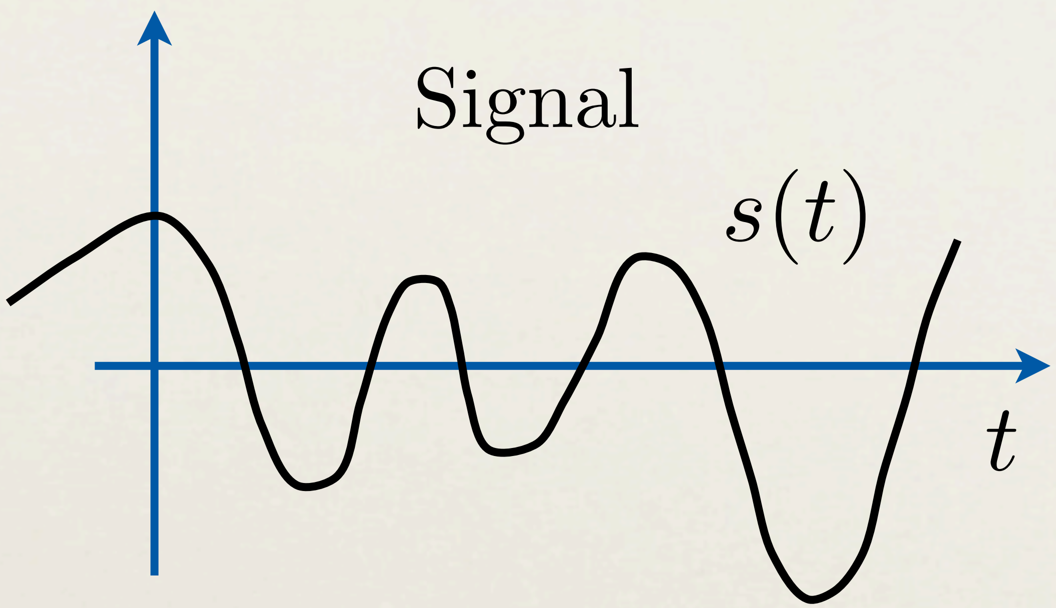


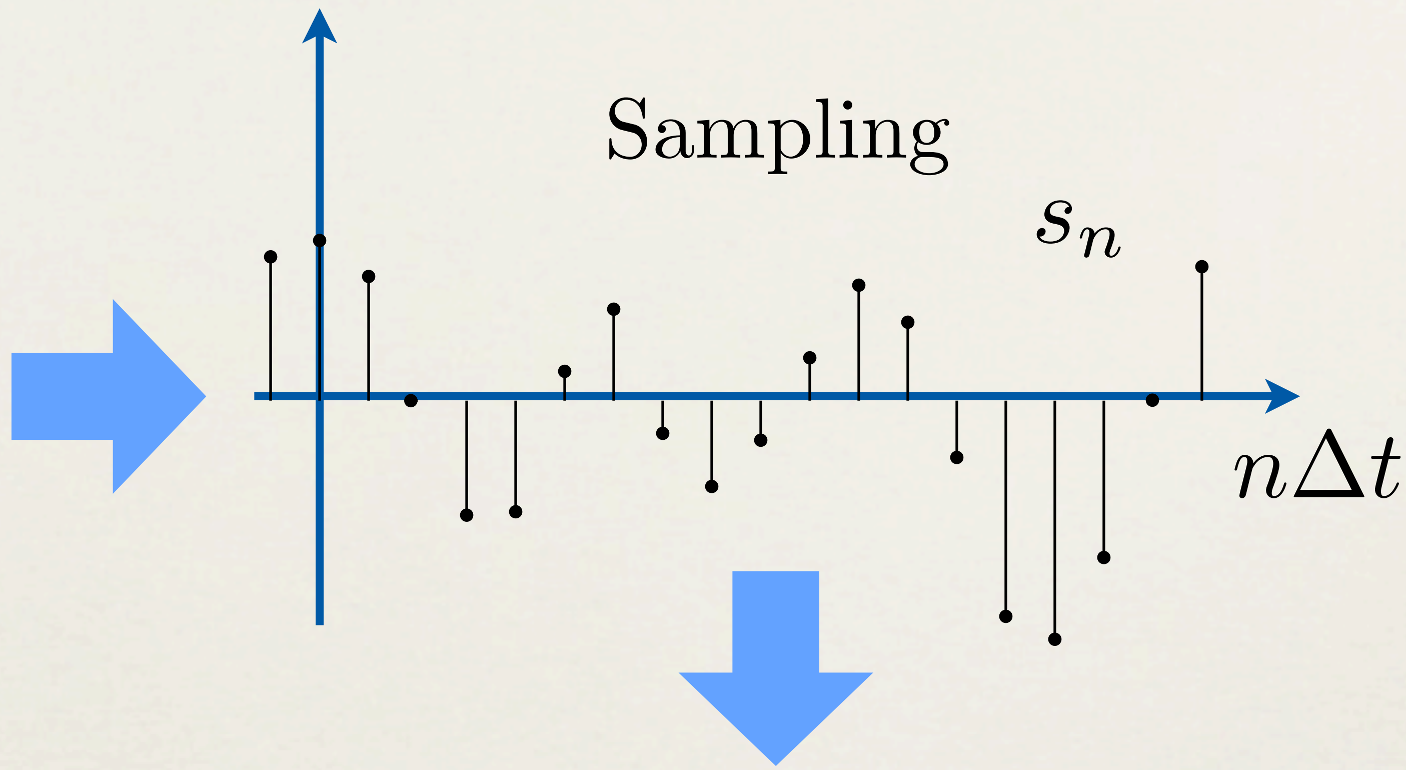




* For which prior?
* For which "Sampling"?
* Which accuracy?

1-bit Sampling?





this talk:

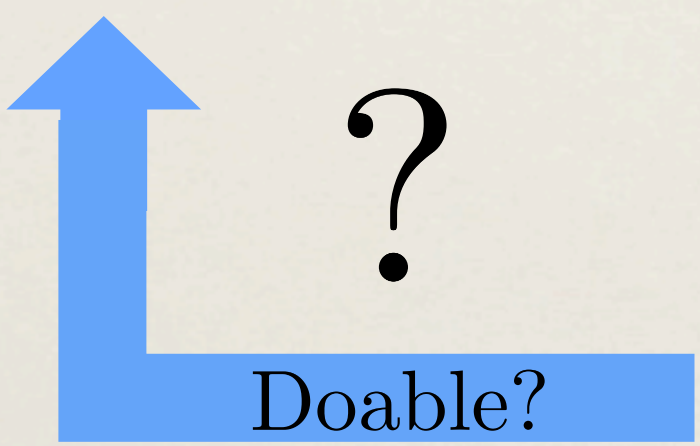
Use random projections

&

Sparsity Prior

&

Compressed Sensing



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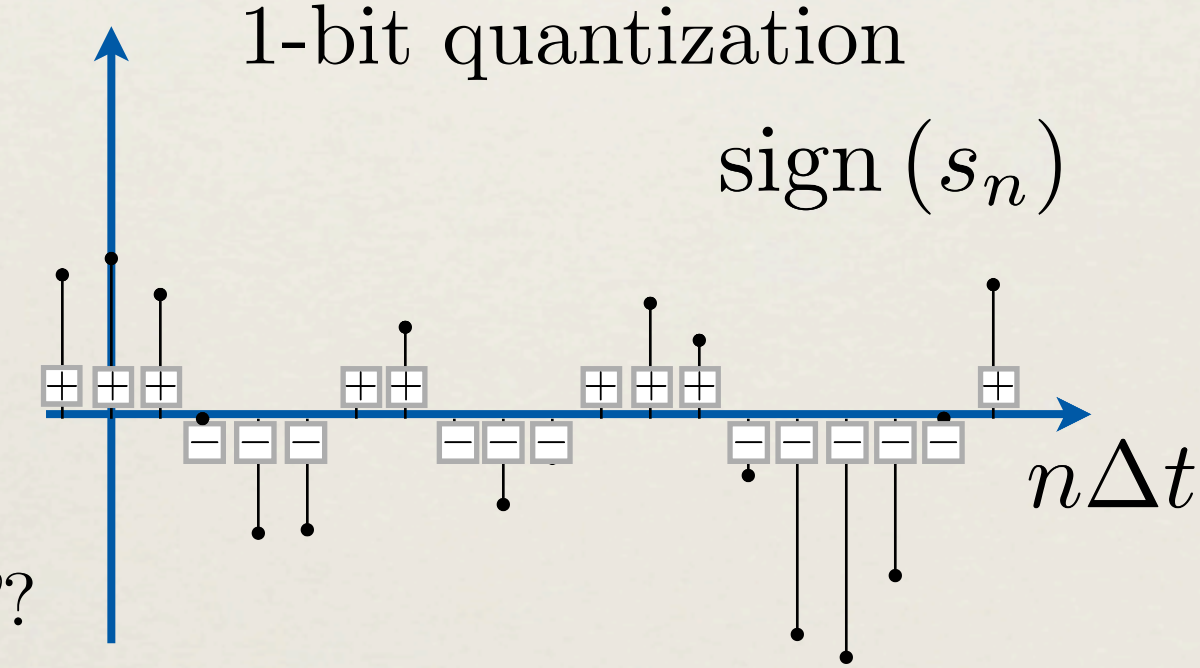
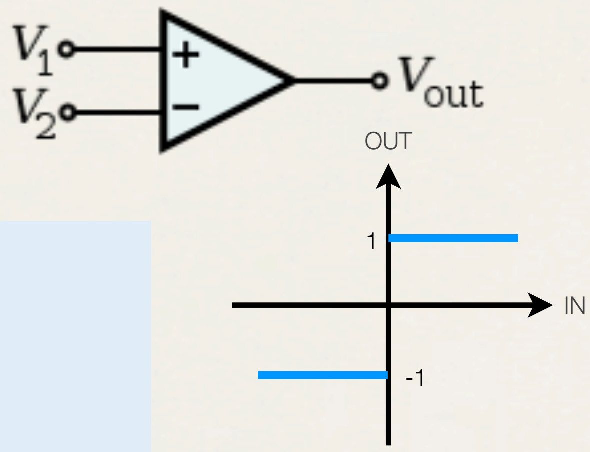
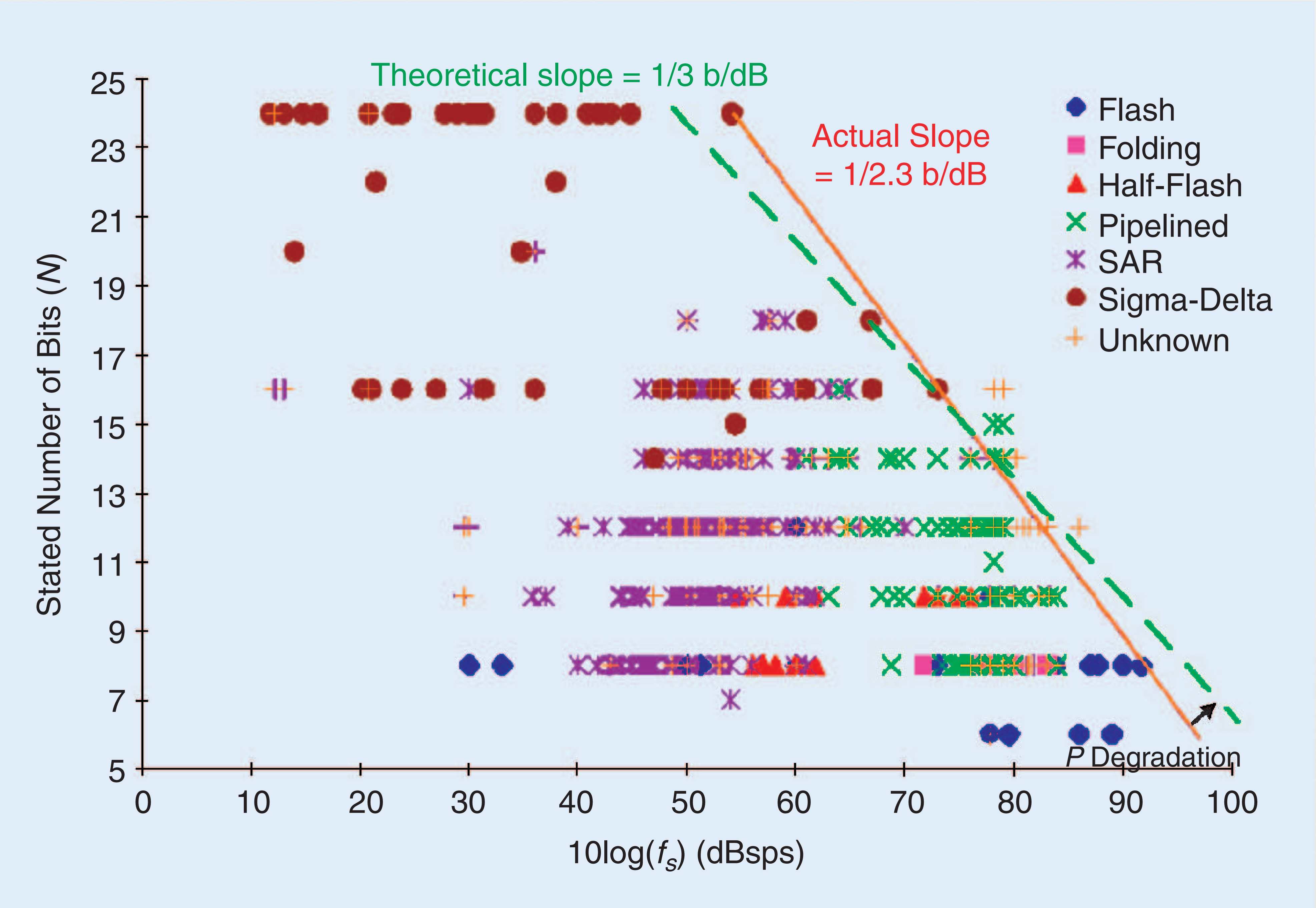


Why 1-bit? Very Fast Quantizers!





[FIG1] **Stated number of bits versus sampling rate.**

[From "Analog-to-digital converters" B. Le, T.W. Rondeau, J.H. Reed, and C.W.Bostian, IEEE Sig. Proc. Magazine, Nov 2005]

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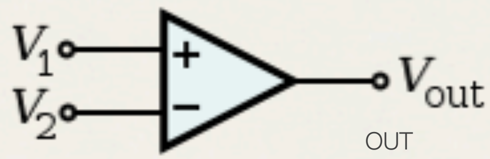

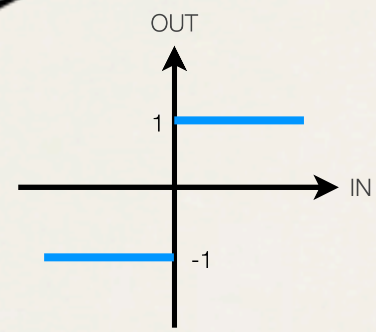

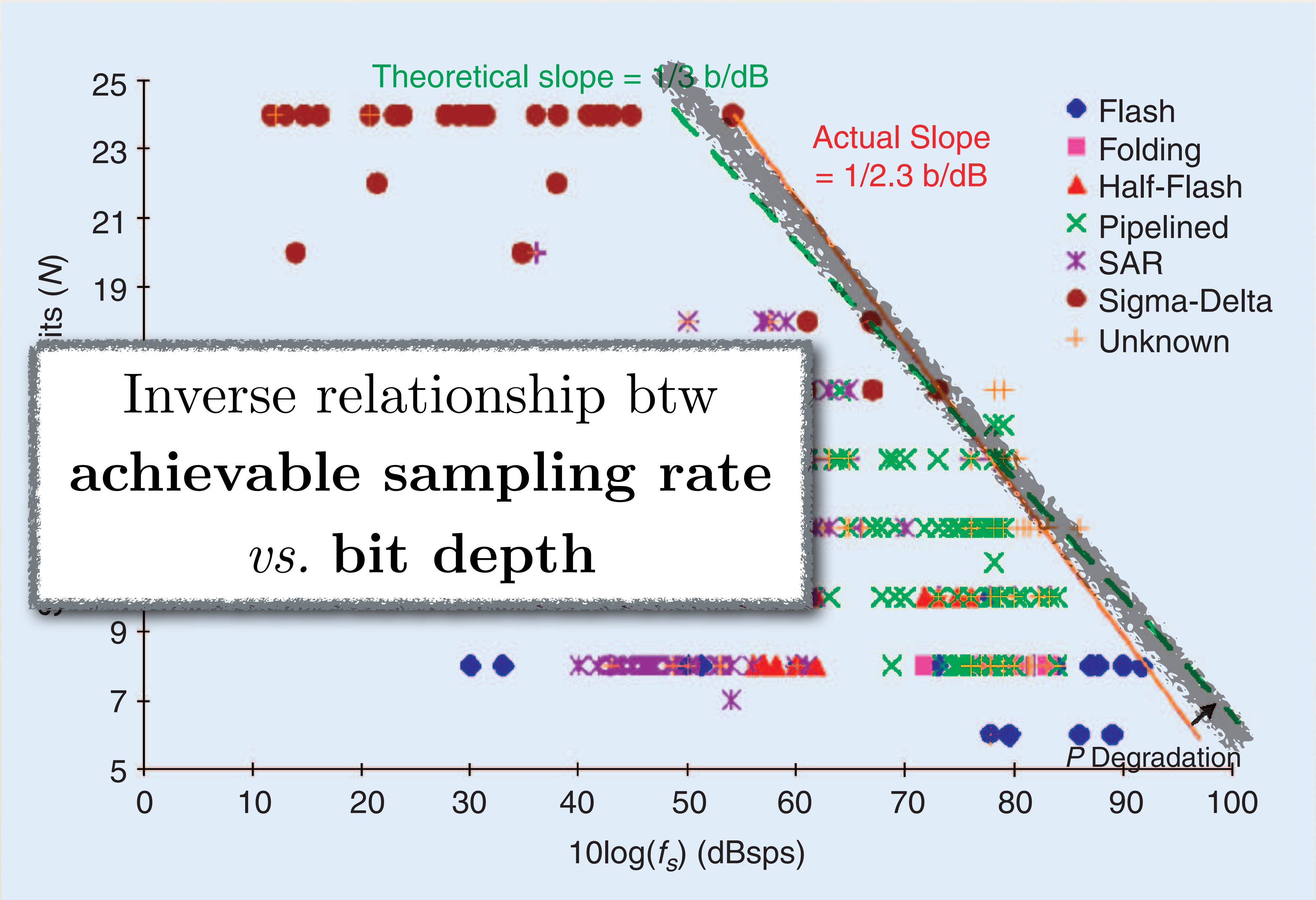

Inverse relationship btw **achievable sampling rate** *vs.* **bit depth**

[FIG1] Stated number of bits versus sampling rate.

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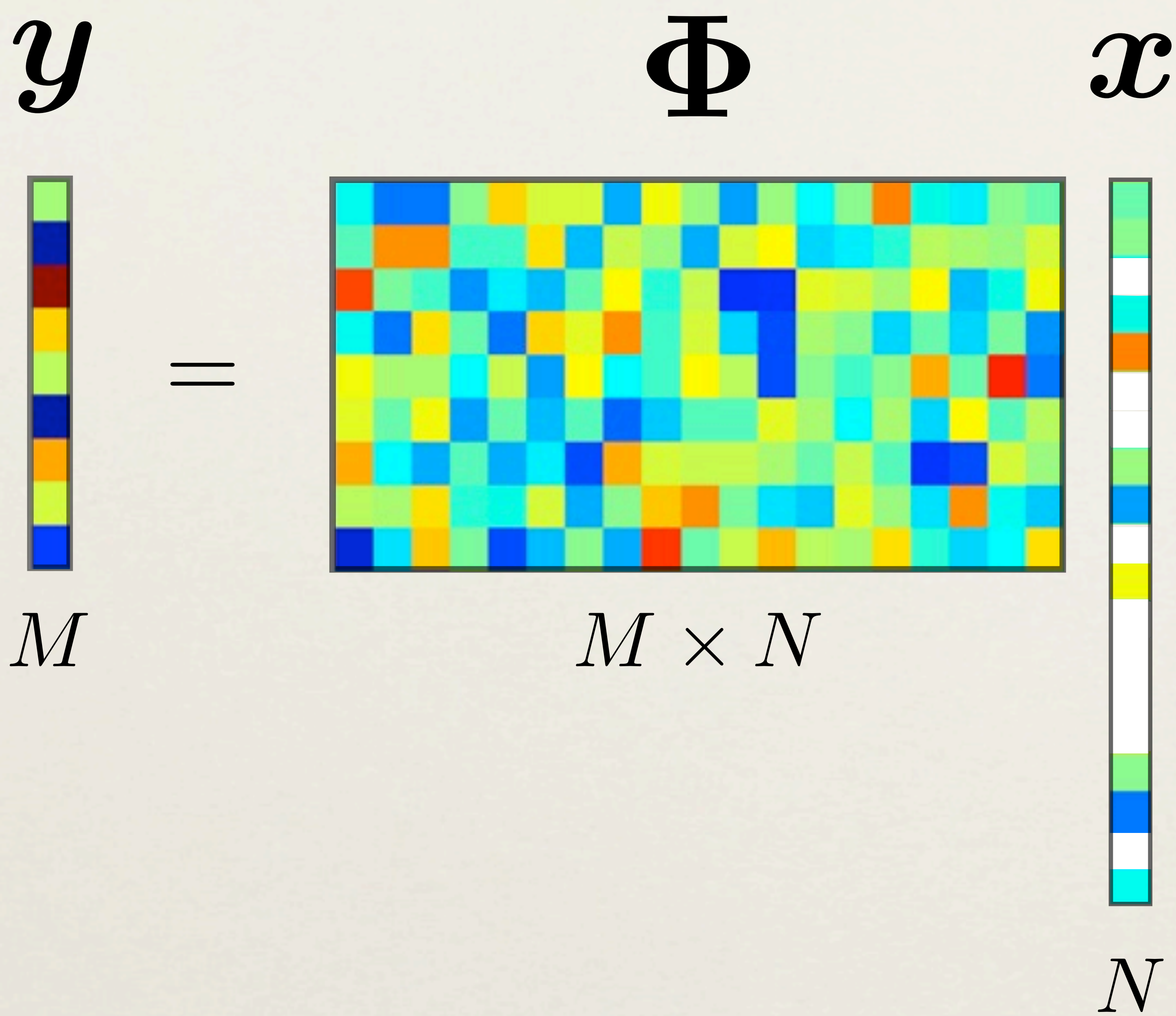
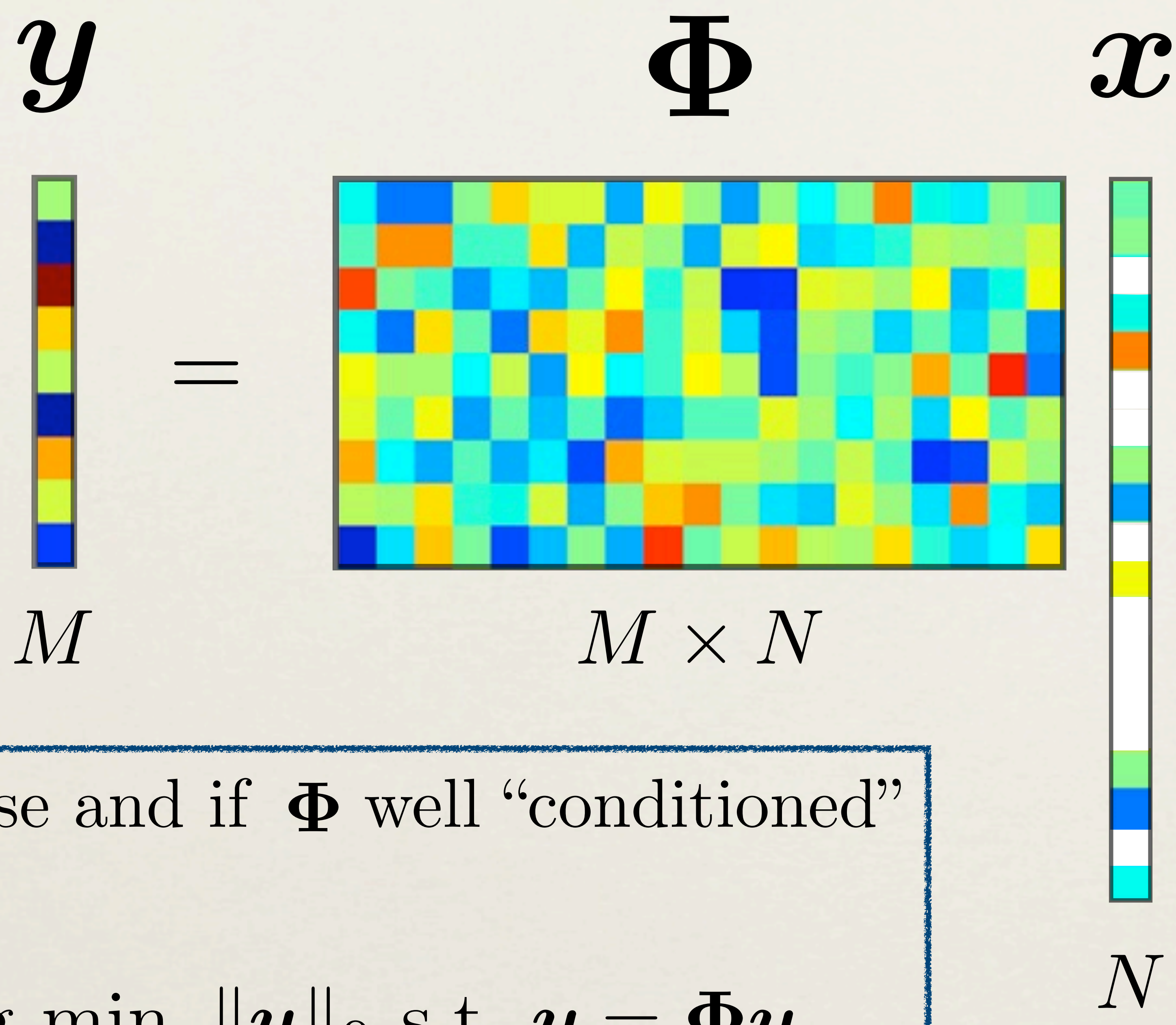
Few slides to understand Compressed Sensing

Just what I need...


$\boldsymbol{y}$
$=$
$\boldsymbol{\Phi}$
$\boldsymbol{x}$
M
$M \times N$
N

Compressed Sensing

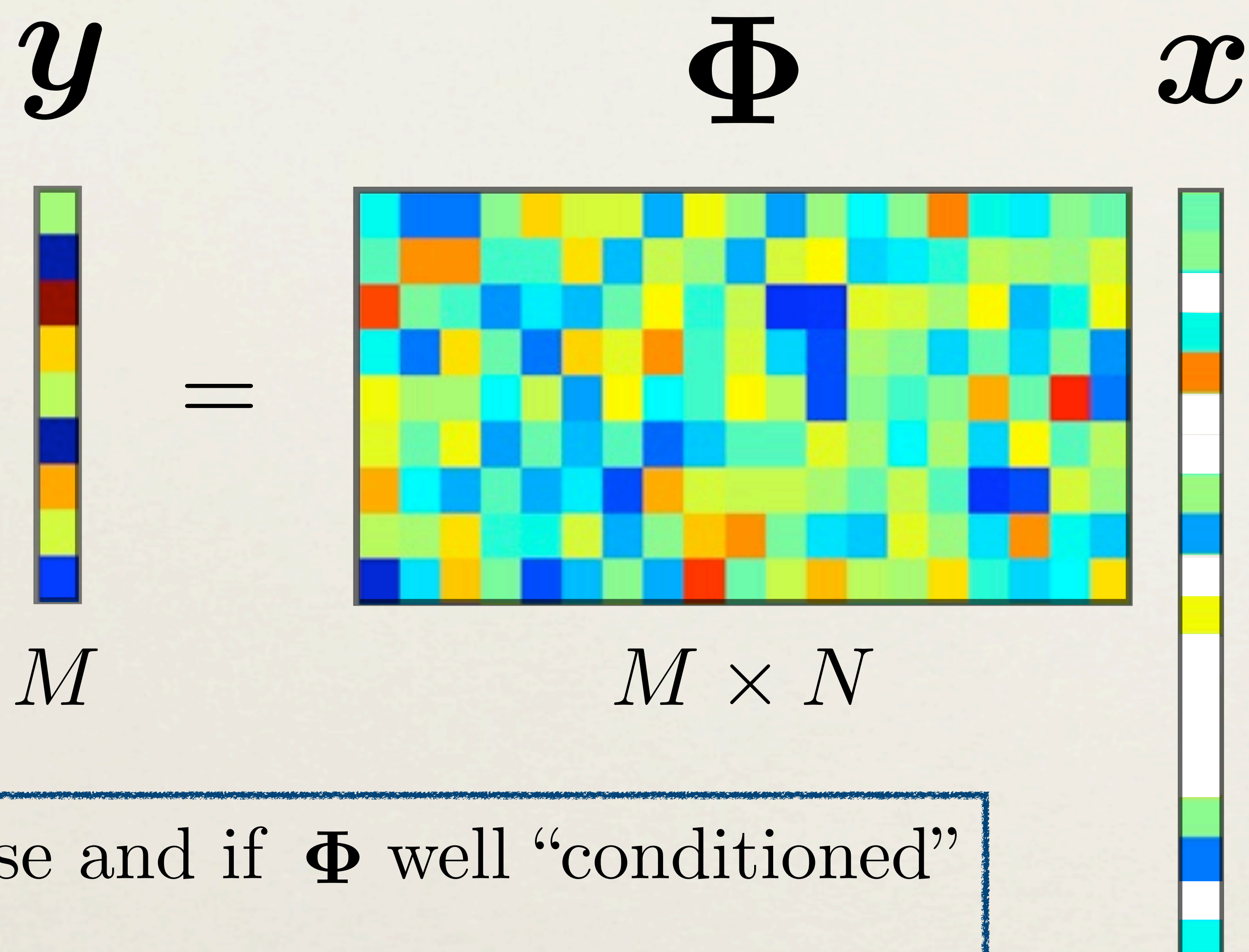


If $\boldsymbol{x}$ is K-sparse and if $\boldsymbol{\Phi}$ well "conditioned" then:

$$\boldsymbol{x} \;=\; \underset{\boldsymbol{u}\in\mathbb{R}^N}{\arg\min}\; \|\boldsymbol{u}\|_0 \text{ s.t. } \boldsymbol{y} = \boldsymbol{\Phi}\boldsymbol{u}$$

$\|\boldsymbol{u}\|_0 = \#\{j : u_j \neq 0\}$

Compressed Sensing



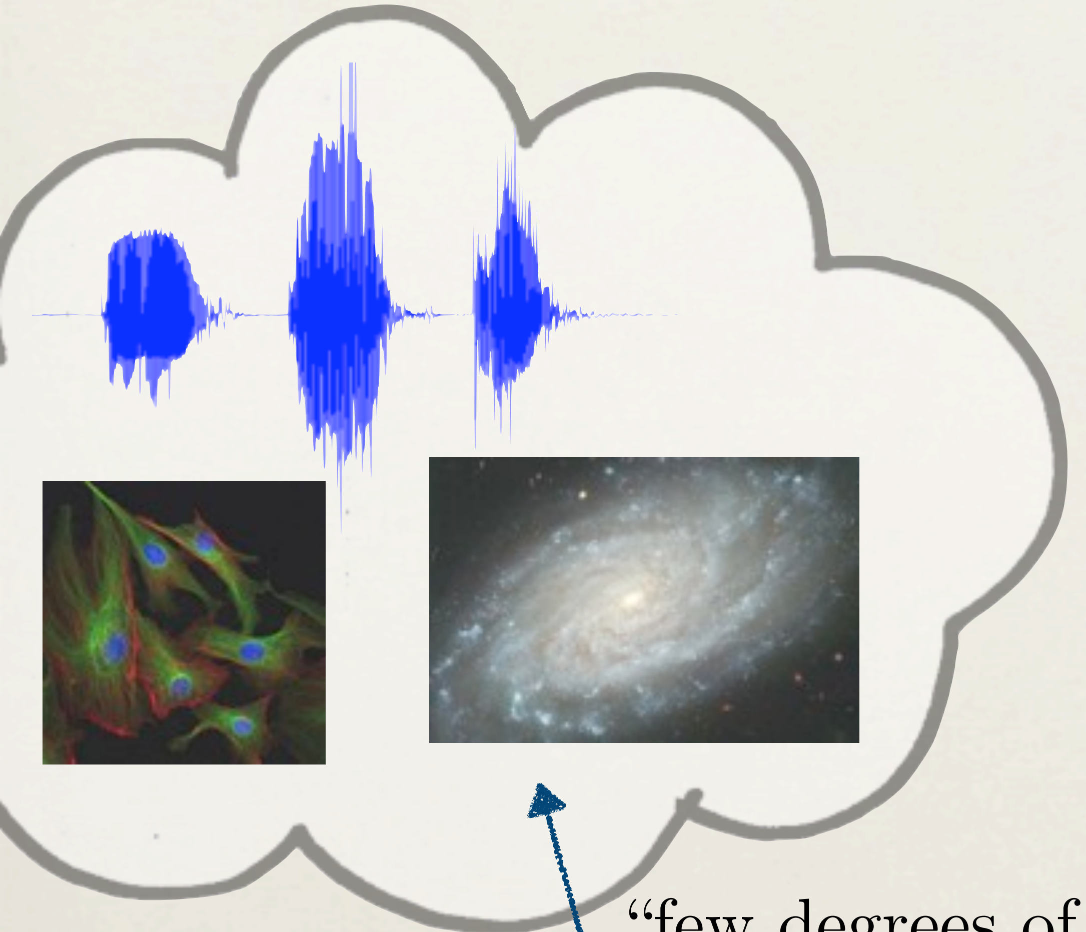
If $\boldsymbol{x}$ is K-sparse and if $\boldsymbol{\Phi}$ well "conditioned"
then:

(relax.)

$$\boldsymbol{x} \;=\; \underset{\boldsymbol{u}\in\mathbb{R}^N}{\arg\min}\ \|\boldsymbol{u}\|_{\cancel{2}\ \mathbf{1}} \ \text{s.t. } \boldsymbol{y} = \boldsymbol{\Phi}\boldsymbol{u}$$

$\|\boldsymbol{u}\|_1 = \sum_j |u_j|$ (Basis Pursuit) [Chen, Donoho, Saunders, 1998]

Compressed Sensing



"few degrees of freedom" law

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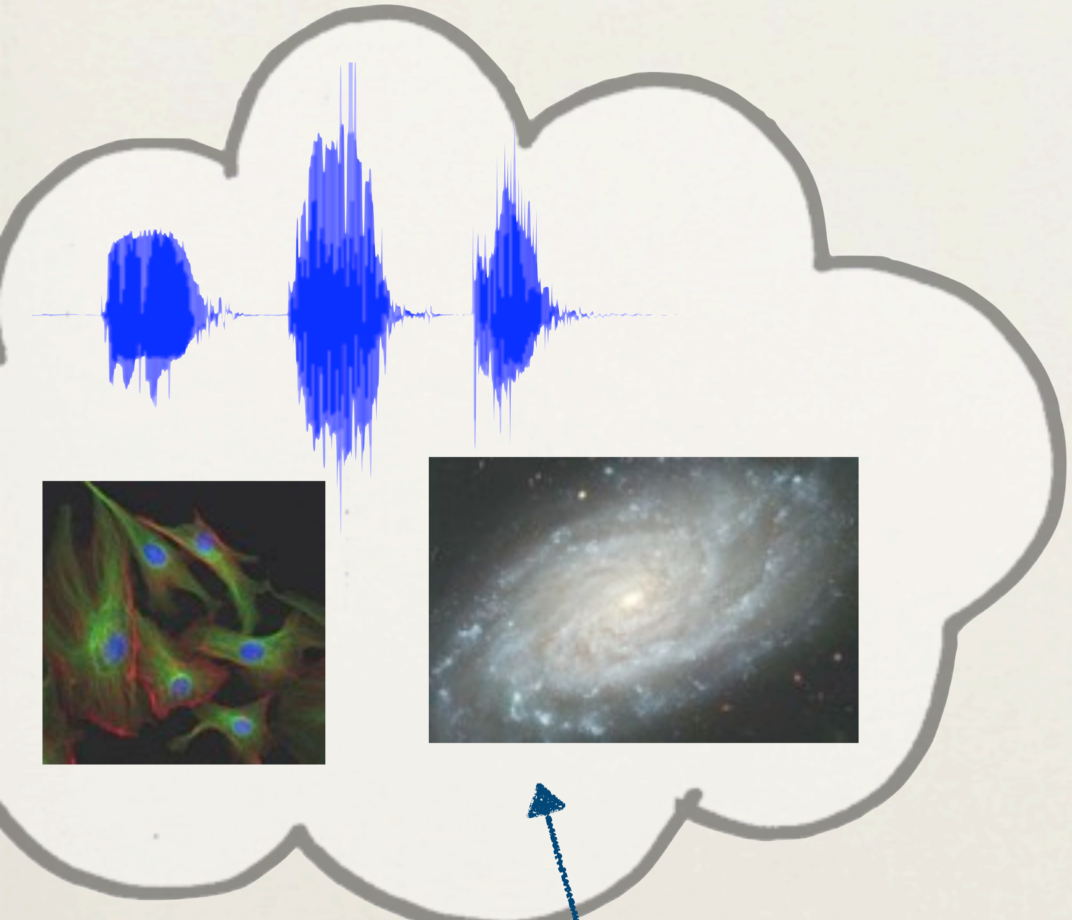

$\exists\, \delta \in (0,1)$ Restricted Isometry Property

$$\sqrt{1-\delta}\,\|\boldsymbol{v}\|_2 \;\leqslant\; \|\boldsymbol{\Phi v}\|_2 \;\leqslant\; \sqrt{1+\delta}\,\|\boldsymbol{v}\|_2$$

for all $2K$ sparse signals $\boldsymbol{v}$.

any subset of $2K$ columns
is an isometry

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if $\delta < \sqrt{2}-1$ [Candes 08]

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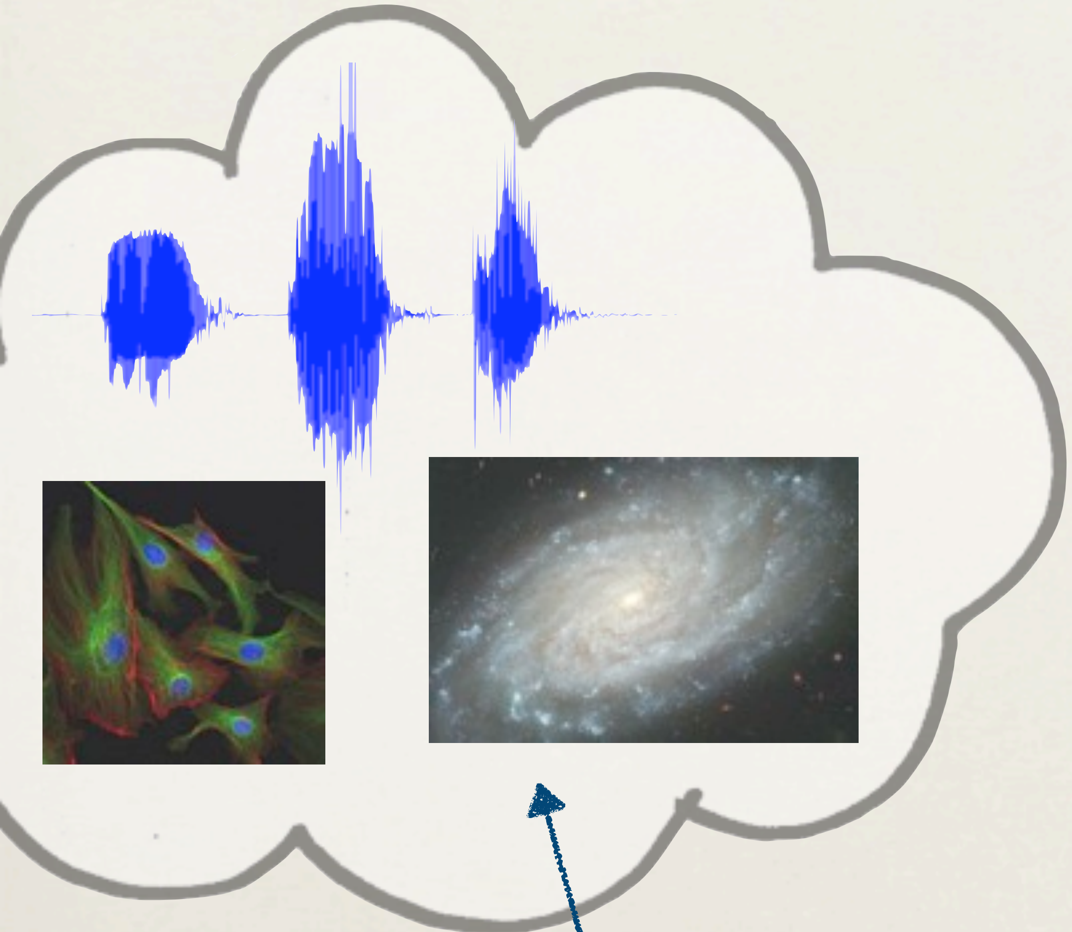

$$M = O(K \ln N/K) \ll N$$
$$\boldsymbol{\Phi} \in \mathbb{R}^{M\times N}, \;\; \Phi_{ij} \sim_{\rm iid} \mathcal{N}(0,1)$$

+ Bernoulli
+ Random Fourier
+ ....

e.g.

<u>Restricted Isometry Property</u>

$\exists\, \delta \in (0,1)$

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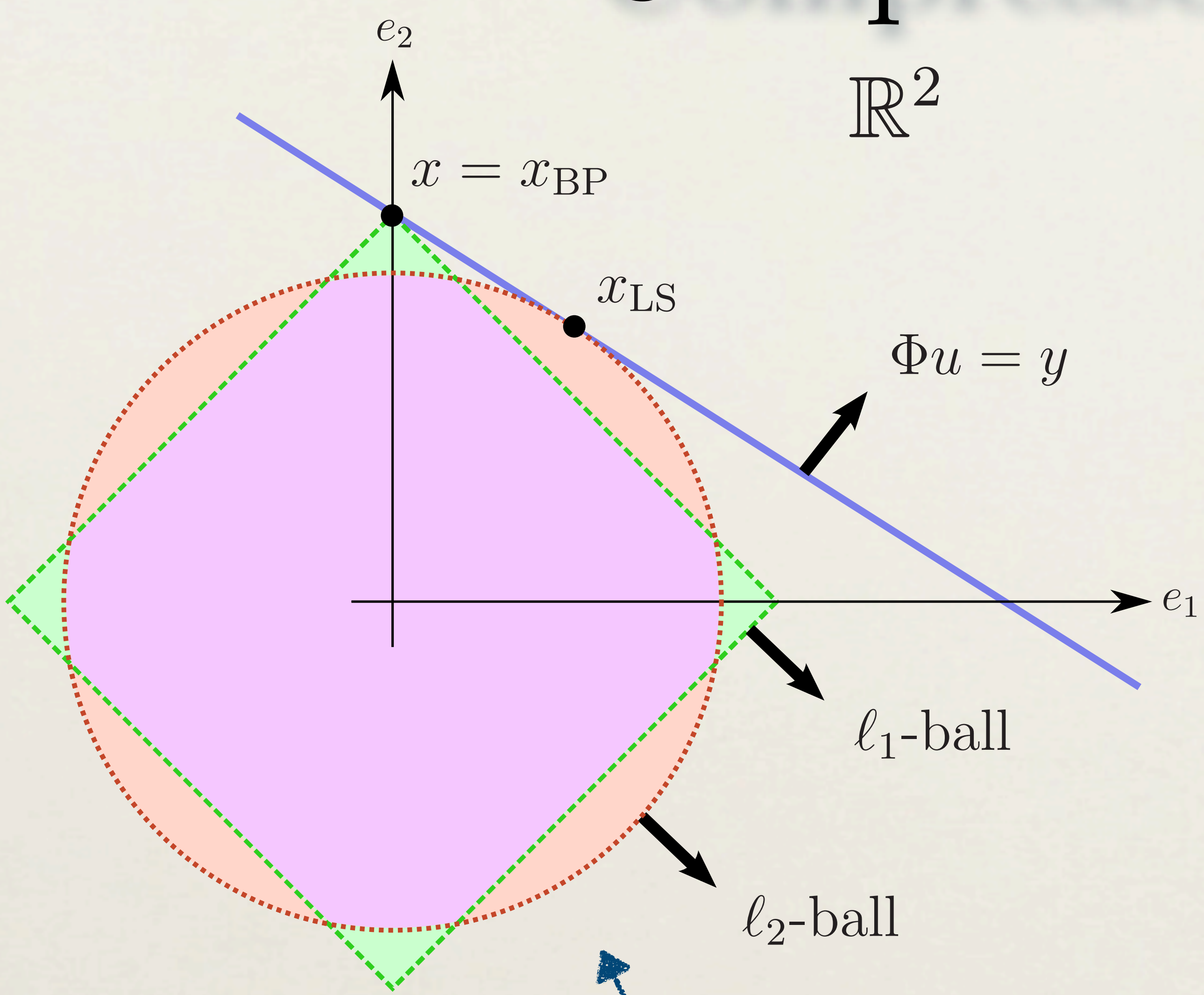

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Compressed Sensing Limits

* CS measurements $\in \mathbb{R}^M$,
  what happens if quantized ? $\mathcal{Q}[\boldsymbol{\Phi x}]$

* "Ok" for high resolution quantization ($\sim$ noise):

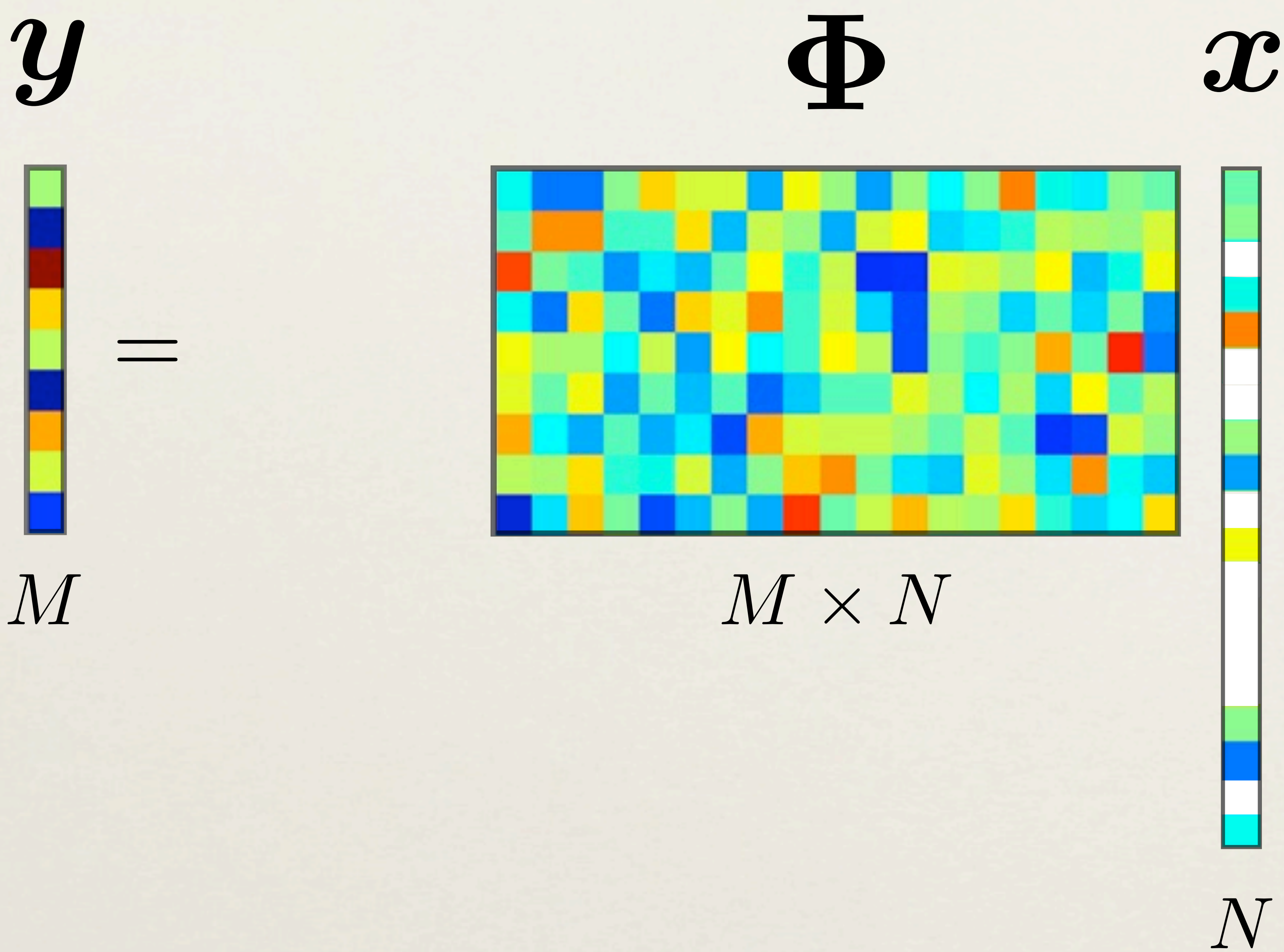
  [Candes, Tao, 2004] [Jacques, Hammond, Fadili 2009, 2011] [Laska et al. 2009] [Dai, Pham, Milenkovic, 2009] ...

$$\|\boldsymbol{x} - \boldsymbol{x}^*\| \leqslant C\,\|\underline{\boldsymbol{y} - \boldsymbol{\Phi x}}\|$$

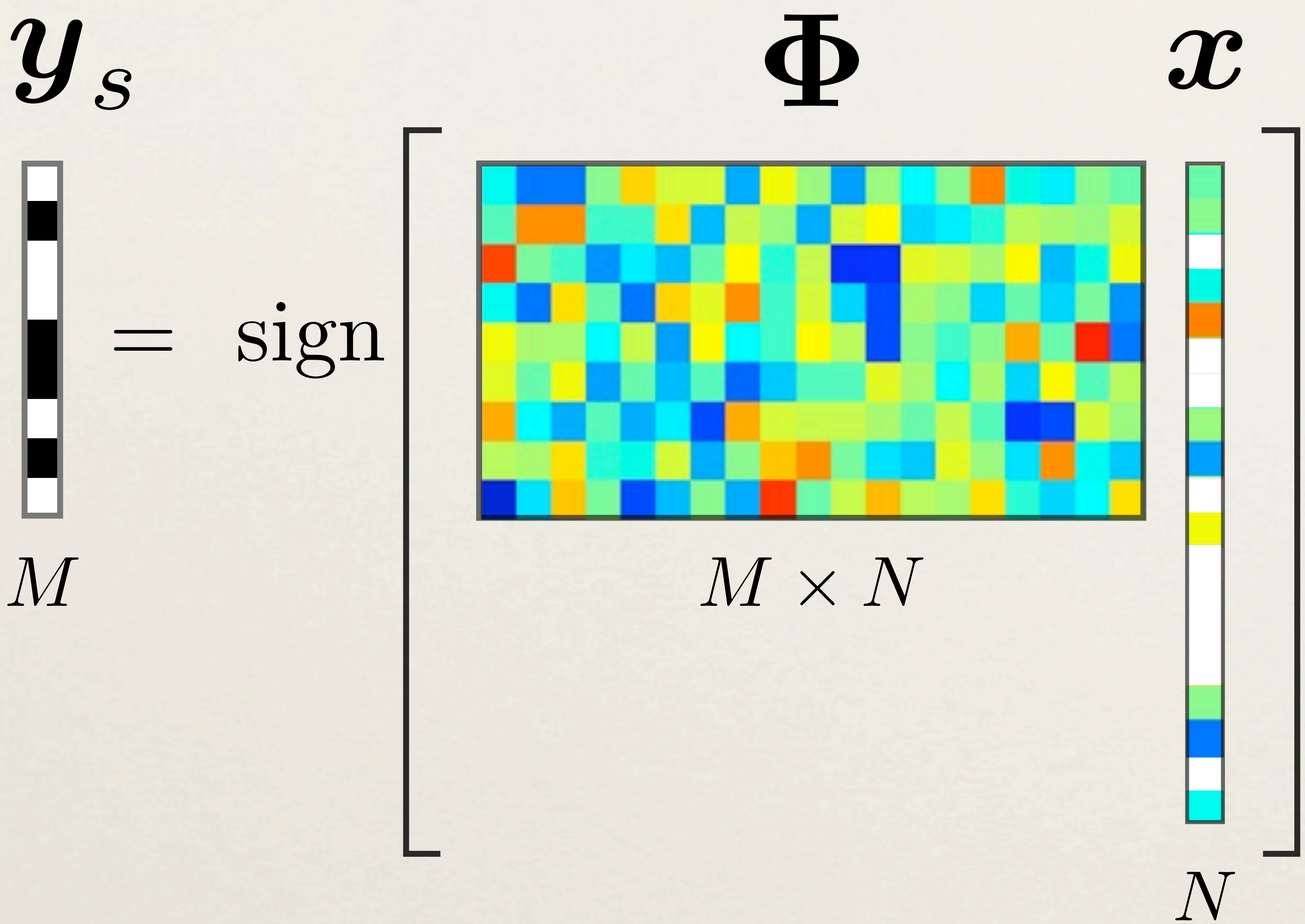
meas. distortion

* But is it still valid for extreme quantization? 1-bit?

Compressed Sensing

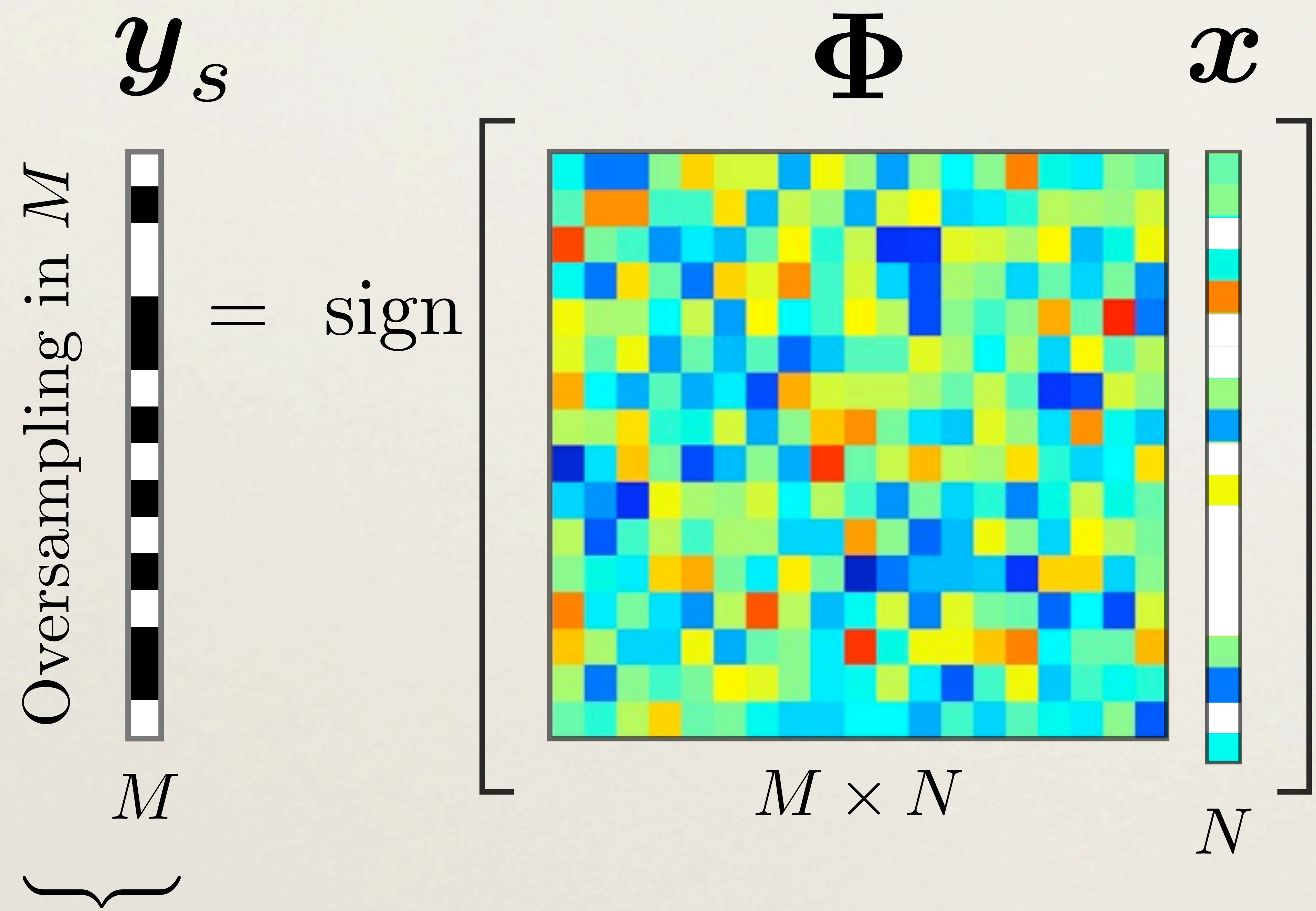


1-bit Compressed Sensing



with: $\mathrm{sign}\, t = \begin{cases} 1 & \text{if } t > 0 \\ -1 & \text{if } t \leqslant 0 \end{cases}$ component-wise

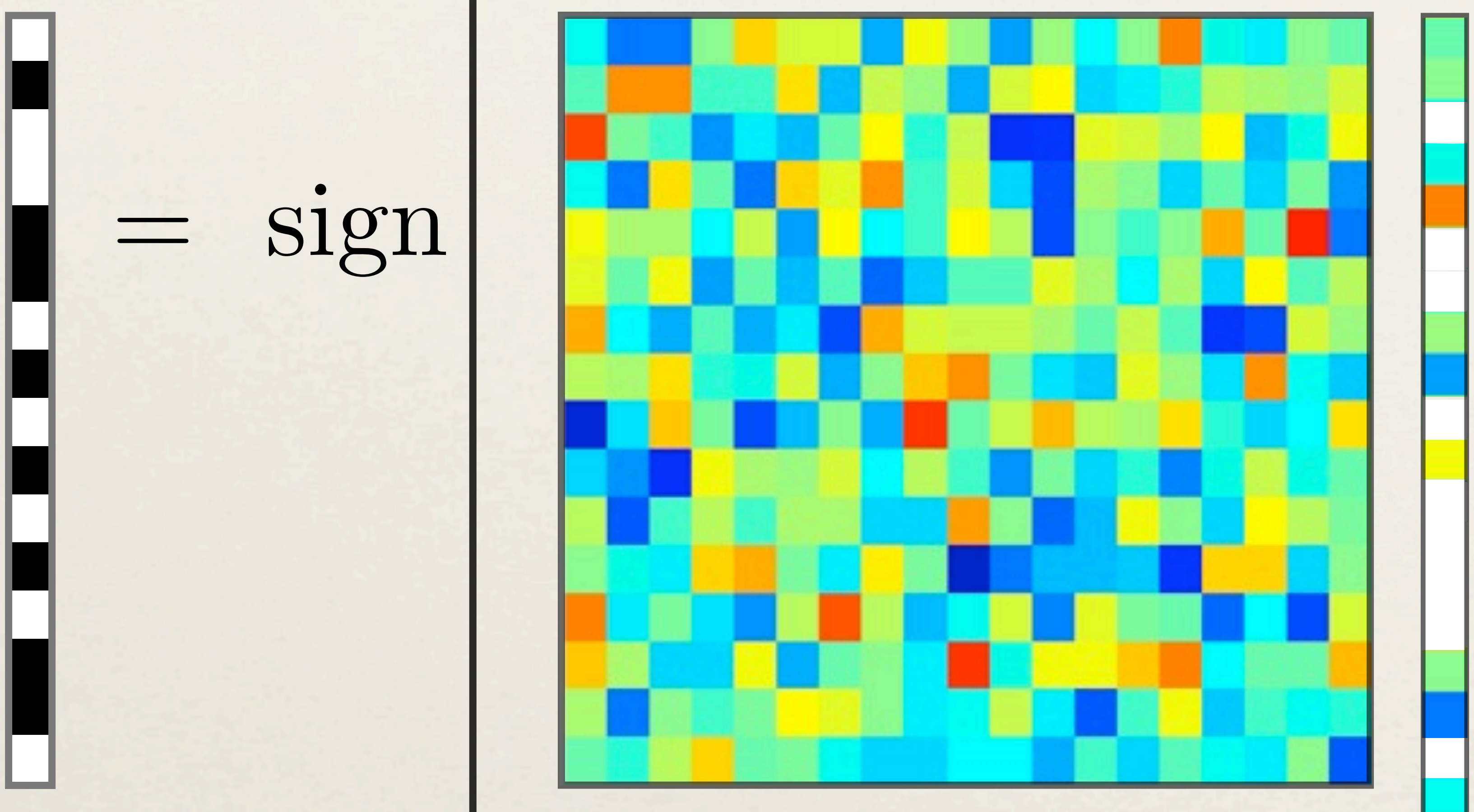
1-bit Compressed Sensing



M-bits! But, which information inside $\boldsymbol{y}_s$?

1-bit ~~Compressed~~ Computational Sensing

$\boxed{\text{bits matter!}}$



$$\underbrace{\boldsymbol{y}_s}_{M\text{-bits!}} = \mathrm{sign}\left[\boldsymbol{\Phi}\,\boldsymbol{x}\right]$$

Oversampling in M

$M \qquad M \times N \qquad N$

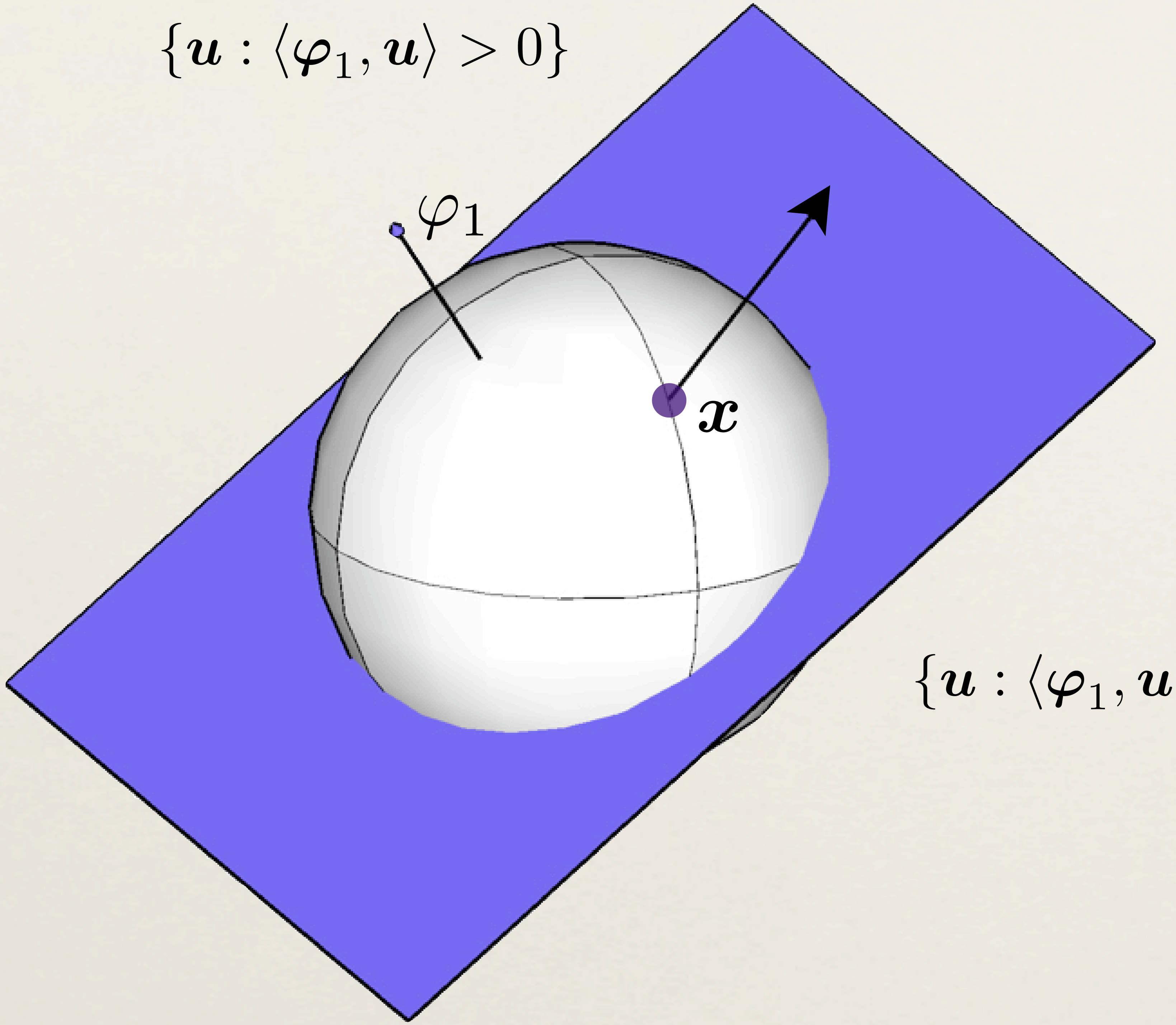
M-bits! But, which information inside $\boldsymbol{y}_s$?

Intuitively ... $\boldsymbol{x}$ on S^2, M vectors $\{\boldsymbol{\varphi}_i : 1 \leqslant i \leqslant M\}$

Intuitively ... $\boldsymbol{x}$ on S^2, M vectors $\{\boldsymbol{\varphi}_i : 1 \leqslant i \leqslant M\}$

1-bit Measurements

$\langle \boldsymbol{\varphi}_1, \boldsymbol{x} \rangle > 0$



[illustration: P. Boufounos]

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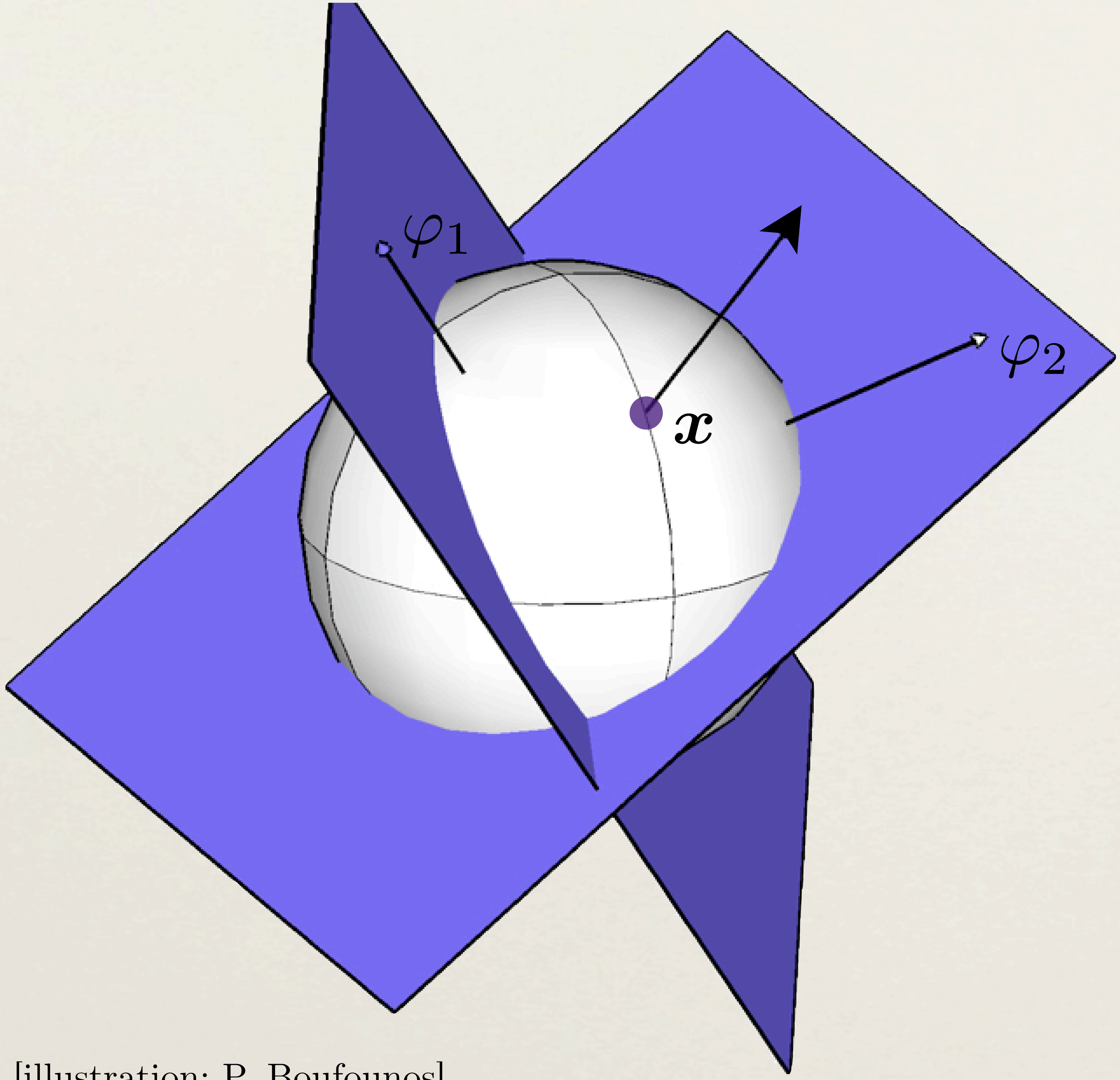

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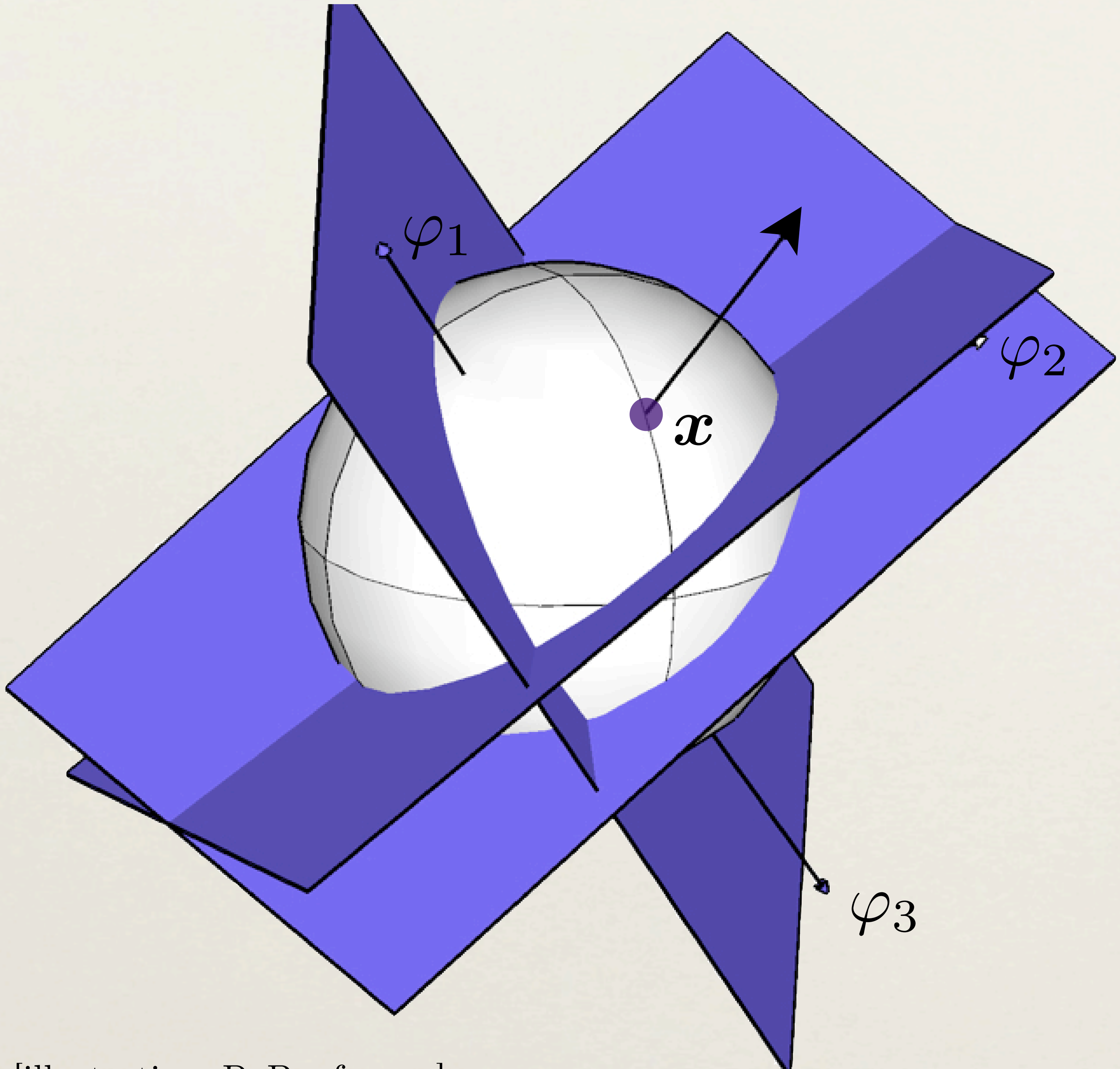

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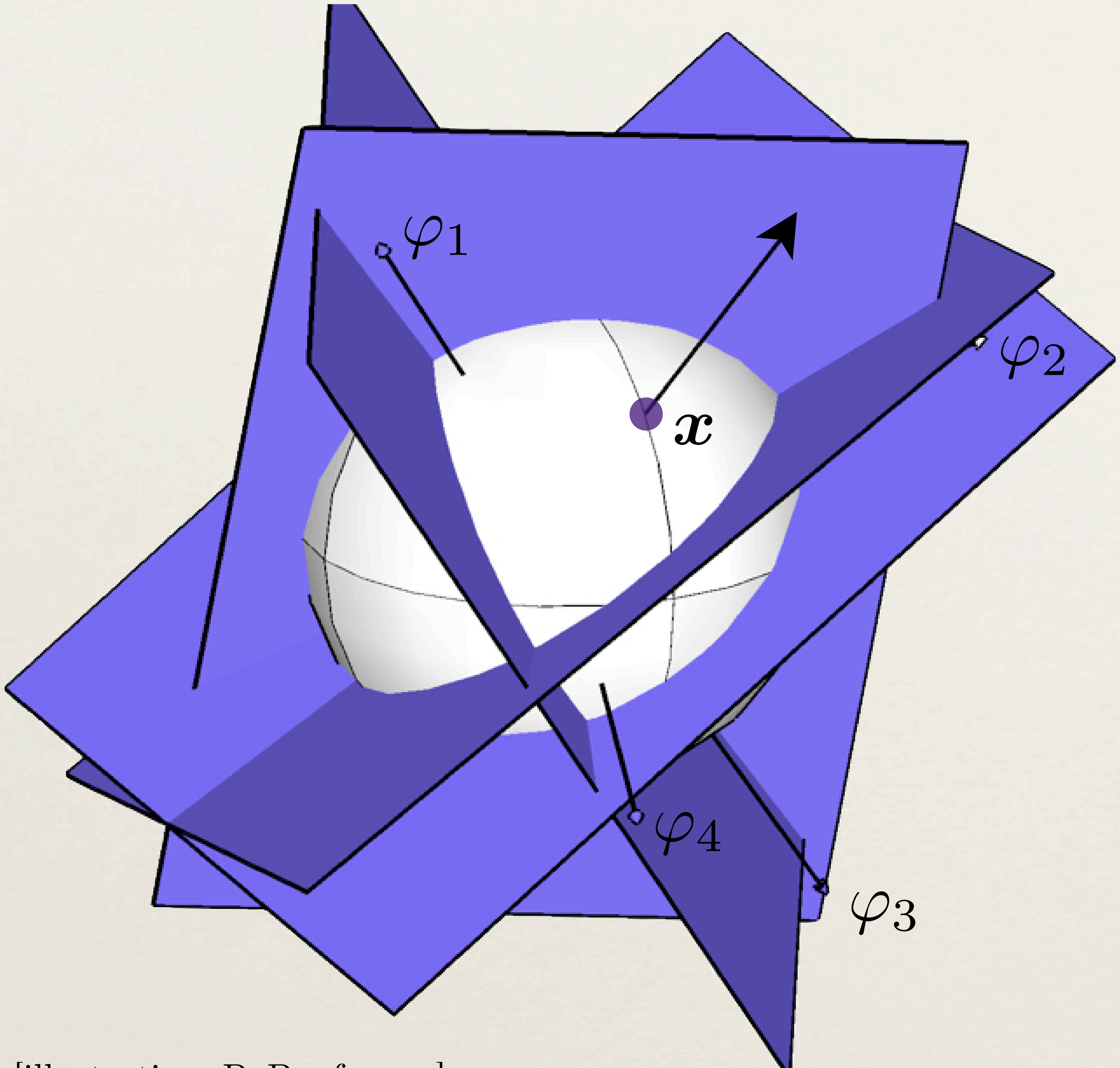
$\langle \boldsymbol{\varphi}_4, \boldsymbol{x} \rangle > 0$



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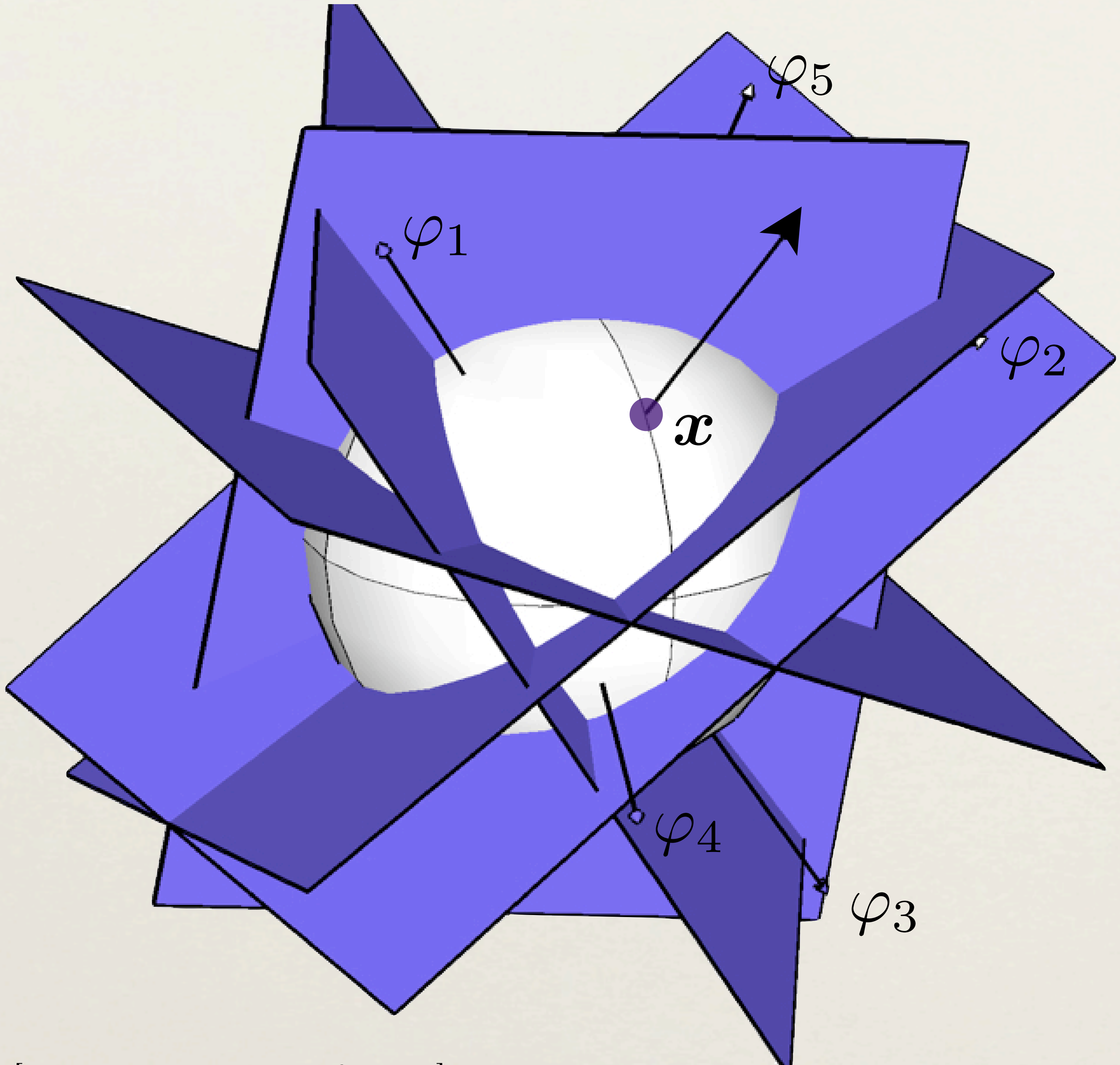
$\langle \boldsymbol{\varphi}_5, \boldsymbol{x} \rangle > 0$



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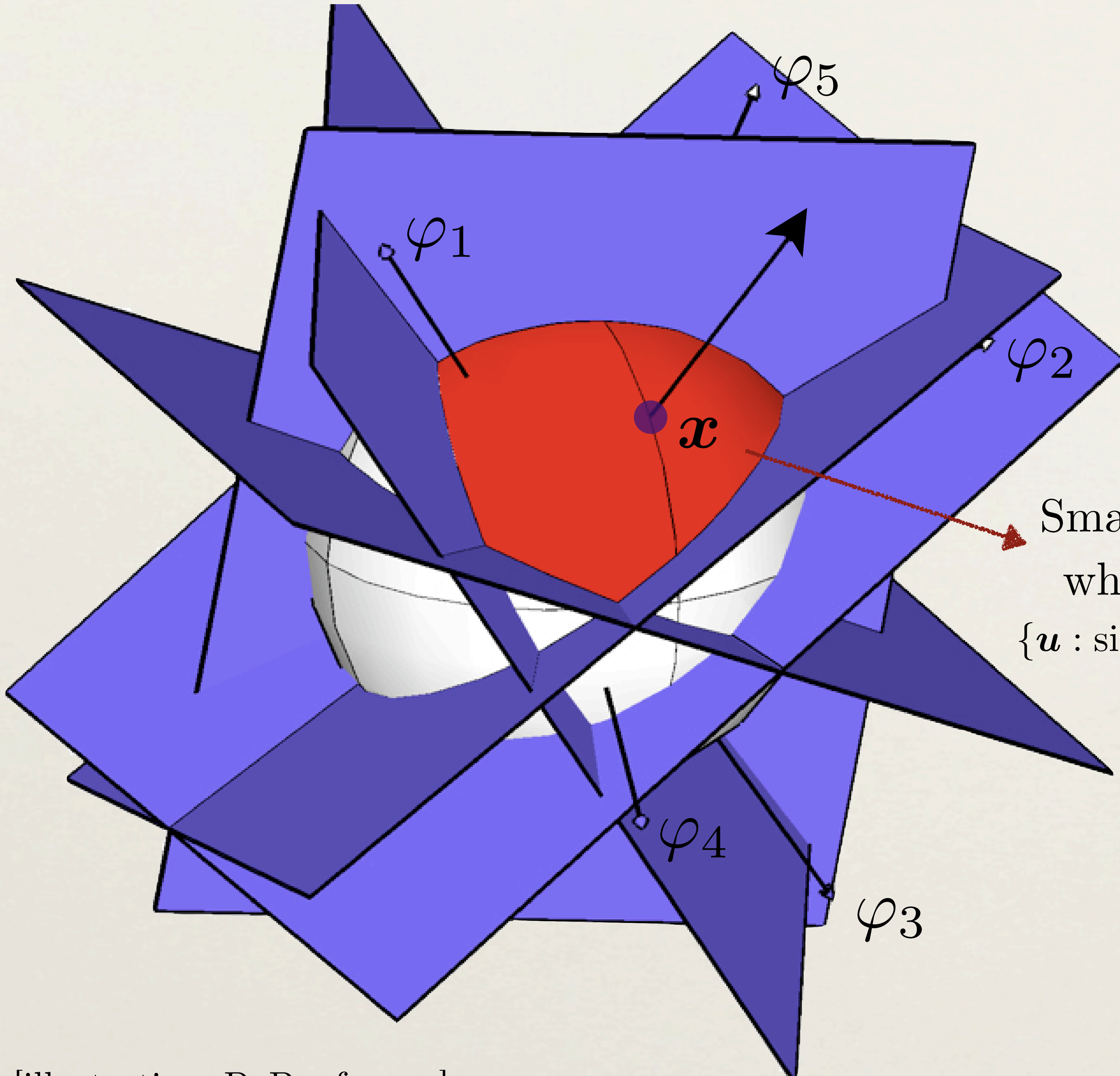

Smaller and smaller when M increases
$\{\boldsymbol{u} : \mathrm{sign}\,(\boldsymbol{\Phi u}) = \mathrm{sign}\,(\boldsymbol{\Phi x})\}$

[illustration: P. Boufounos]

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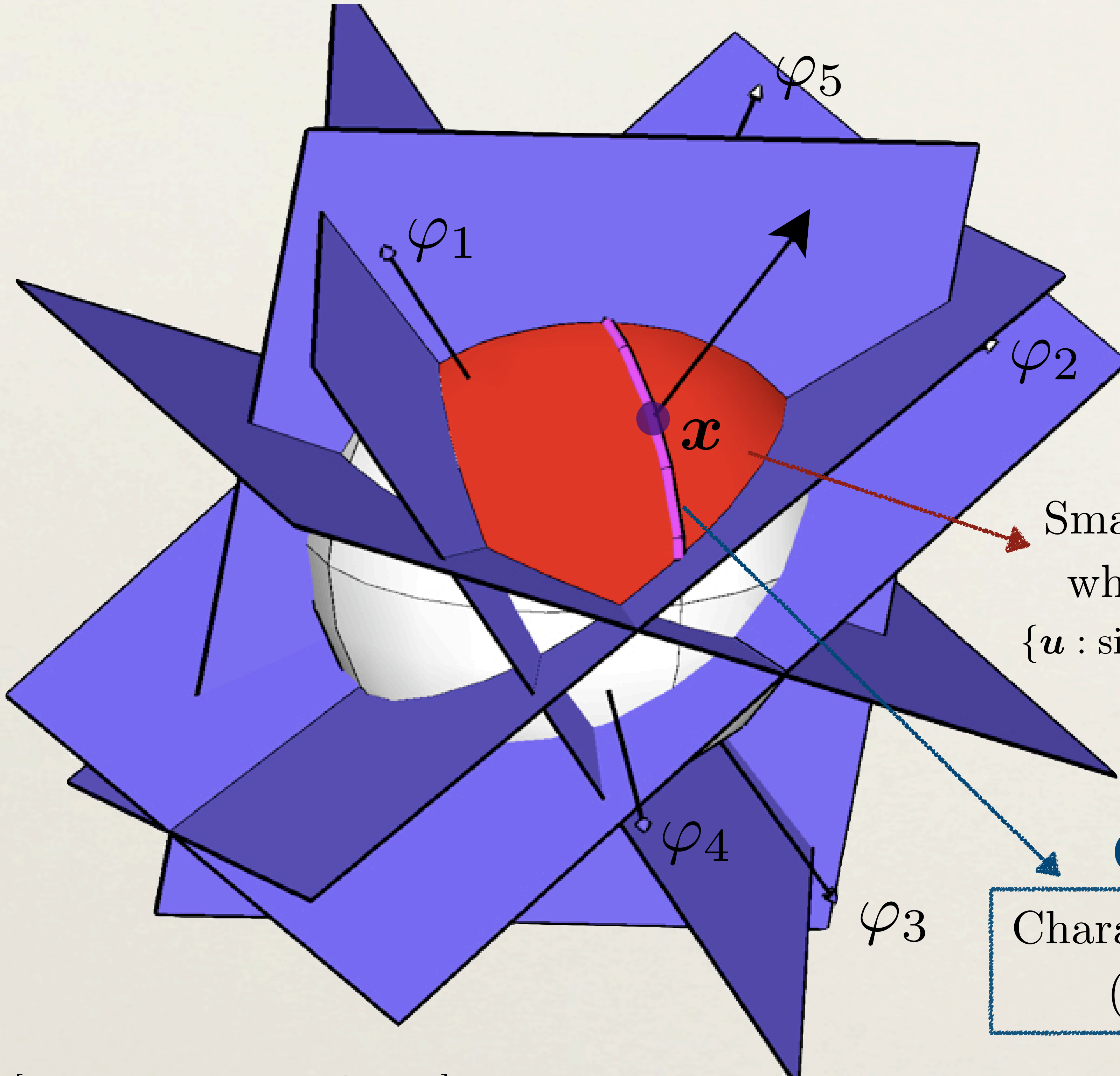

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Objective

Characterize the width (particular $\boldsymbol{x}$)

[illustration: P. Boufounos]

Starting point: Hamming/Angle Concentration

* Metrics of interest:

$$d_H(\boldsymbol{u}, \boldsymbol{v}) = \tfrac{1}{M} \textstyle\sum_i (u_i \oplus v_i) \quad \text{(norm. Hamming)}$$

$$d_{\rm ang}(\boldsymbol{x}, \boldsymbol{s}) = \tfrac{1}{\pi} \arccos(\langle \boldsymbol{x}, \boldsymbol{s} \rangle) \quad \text{(norm. angle)}$$

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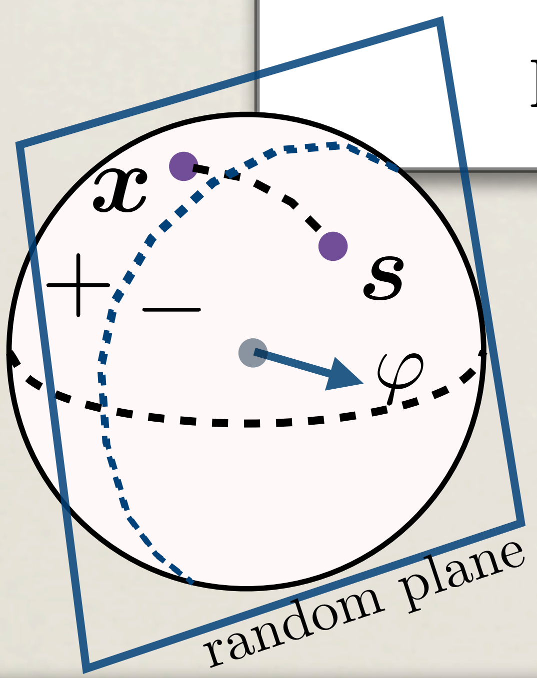
- Known fact: if $\boldsymbol{\Phi} \sim \mathcal{N}^{M\times N}(0,1)$ [*e.g.*, Goemans, Williamson 1995]

Let $\boldsymbol{\Phi} \sim \mathcal{N}^{M\times N}(0,1)$, $A(\cdot) = \mathrm{sign}\,(\boldsymbol{\Phi}\,\cdot) \in \{-1,1\}^M$ and $\epsilon > 0$.
For any $\boldsymbol{x},\boldsymbol{s} \in S^{N-1}$, we have

$$\mathbb{P}_{\boldsymbol{\Phi}}\Big[\,\big|\,d_H\big(A(\boldsymbol{x}),A(\boldsymbol{s})\big)\;-\;d_{\rm ang}(\boldsymbol{x},\boldsymbol{s})\big|\;\leqslant\;\epsilon\,\Big]\;\geqslant\;1-2\,e^{-2\epsilon^2 M}.$$



Thanks to $A(\cdot)$, Hamming distance concentrates around vector angles!

Binary ϵ Stable Embedding (BϵSE)

A mapping $A : \mathbb{R}^N \to \mathcal{B}^M$ is a **binary ϵ-stable embedding** (BϵSE) of order K for sparse vectors if

$$d_{\rm ang}(\boldsymbol{x}, \boldsymbol{s}) - \epsilon \leqslant d_H(A(\boldsymbol{x}), A(\boldsymbol{s})) \leqslant d_{\rm ang}(\boldsymbol{x}, \boldsymbol{s}) + \epsilon$$

for all $\boldsymbol{x}, \boldsymbol{s} \in S^{N-1}$ with $\boldsymbol{x} \pm \boldsymbol{s}$ K-sparse.

kind of "binary restricted (quasi) isometry"

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* *Corollary*: for any algorithm with output $\boldsymbol{x}^*$ jointly K-sparse and consistent (*i.e.*, $A(\boldsymbol{x}^*) = A(\boldsymbol{x})$),

$$d_{\rm ang}(\boldsymbol{x}, \boldsymbol{x}^*) \leqslant 2\epsilon!$$

* If limited binary noise, $d_{\rm ang}$ still bounded
* If not exactly sparse signals (but almost), $d_{\rm ang}$ still bounded

BϵSE existence? Yes!

Let $\mathbf{\Phi} \sim \mathcal{N}^{M\times N}(0,1)$, fix $0 \leqslant \eta \leqslant 1$ and $\epsilon > 0$. If

$$M \;\geqslant\; \tfrac{4}{\epsilon^2}\big(K\,\ln(N) + 2K\,\ln(\tfrac{50}{\epsilon}) + \ln(\tfrac{2}{\eta})\big),$$

then $\mathbf{\Phi}$ is a BϵSE with $\Pr > 1-\eta$.

$$M = O(\epsilon^{-2} K \ln N)$$

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Proof sketch:

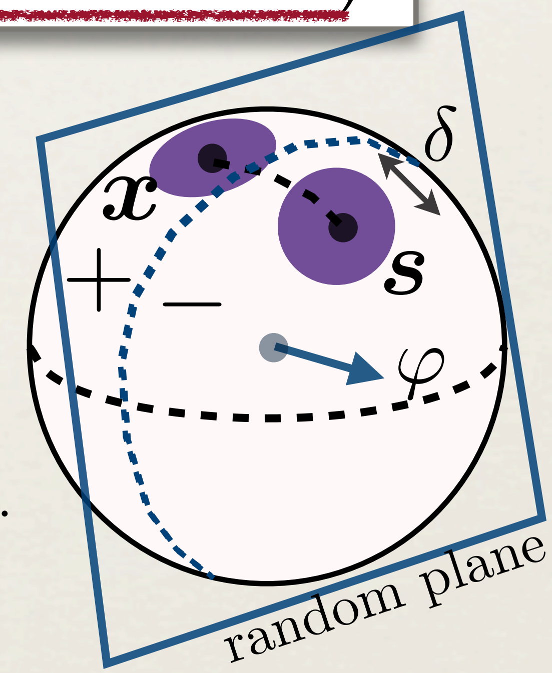
1) Generalize

$$\mathbb{P}_{\boldsymbol{\Phi}}\big[\,\big|d_H\big(A(\boldsymbol{x}),A(\boldsymbol{s})\big) \;-\; d_{\rm ang}(\boldsymbol{x},\boldsymbol{s})\big| \;\leqslant\; \epsilon\,\big] \;\geqslant\; 1-2\,e^{-2\epsilon^2 M}.$$

to

$$\mathbb{P}_{\boldsymbol{\Phi}}\Big[\,\big|d_H\big(A(\boldsymbol{u}),A(\boldsymbol{v})\big) \;-\; d_{\rm ang}(\boldsymbol{x},\boldsymbol{s})\big| \;\leqslant\; \epsilon + (\tfrac{\pi}{2}D)^{1/2}\,\delta\,\Big] \;\geqslant\; 1-2\,e^{-2\epsilon^2 M}.$$

for $\boldsymbol{u},\boldsymbol{v}$ in a D-dimensional neighborhood of width δ around $\boldsymbol{x}$ and $\boldsymbol{s}$ resp.



2) Covers the space of "K-sparse signal pairs" in $\mathbb{R}^N$ by

$$O\big(\tbinom{N}{K}\delta^{-2K}\big) = O\big((\tfrac{eN}{K\delta^2})^K\big) \text{ neighborhoods.}$$

3) Apply Point 1 with union bound, and "stir until the proof thickens"

Hope for better bounds? ... Limited

BϵSE consistency "width":

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Lower bound: Worst case distance btw two unit K-sparse vectors:

$$\epsilon_{\text{worst}} \;= \Omega\big(K/M\big)$$

[P. Boufounos]

1-bit CS Reconstructions ?

Numerical Reconstructions:

* [Boufounos, Baraniuk 2008]

$$\boxed{\boldsymbol{x}^* = \arg\min_{\boldsymbol{u}} \|\boldsymbol{u}\|_1 \quad \text{s.t.} \quad \mathrm{diag}(A(\boldsymbol{x}))\,\Phi\boldsymbol{u} > 0 \quad \text{and} \quad \|\boldsymbol{u}\|_2 = 1}$$

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- **B**inary **I**terative **H**ard **T**hresholding (BIHT):

Given $\boldsymbol{y}_s = A(\boldsymbol{x})$ and K, set $l = 0$, $\boldsymbol{x}^0 = 0$: ($\tau > 0$ controls gradient descent)

$$\boxed{\begin{aligned} \boldsymbol{a}^{l+1} &= \boldsymbol{x}^l + \tfrac{\tau}{2}\Phi^T\big(\boldsymbol{y}_s - A(\boldsymbol{x}^l)\big), \\ \boldsymbol{x}^{l+1} &= \eta_K(\boldsymbol{a}^{l+1}), \quad l \leftarrow l+1 \end{aligned}}$$

("gradient" towards consistency)

(proj. K-sparse signal set)

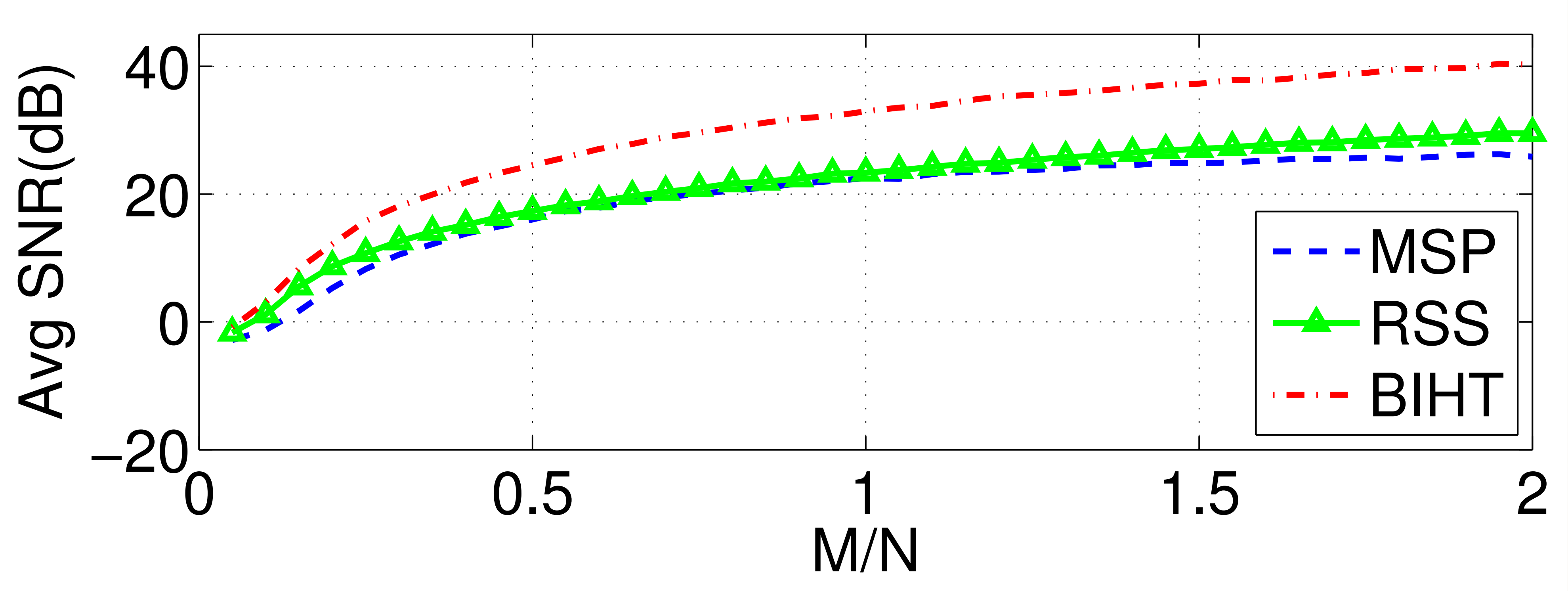
Stop when $d_H(\boldsymbol{y}_s, A(\boldsymbol{x}^{l+1})) = 0$ or $l =$ max. iter.

with $\eta_K(\boldsymbol{u})$ = best K-term approximation of $\boldsymbol{u}$

minimizes $\mathcal{J}(\boldsymbol{x}) = \|[\mathrm{diag}(\boldsymbol{y}_s)(\Phi\boldsymbol{x})]_-\|_1$, with $(\lambda)_- = (\lambda - |\lambda|)/2$

(connections with ML hinge loss, 1-bit classification)

Simulations



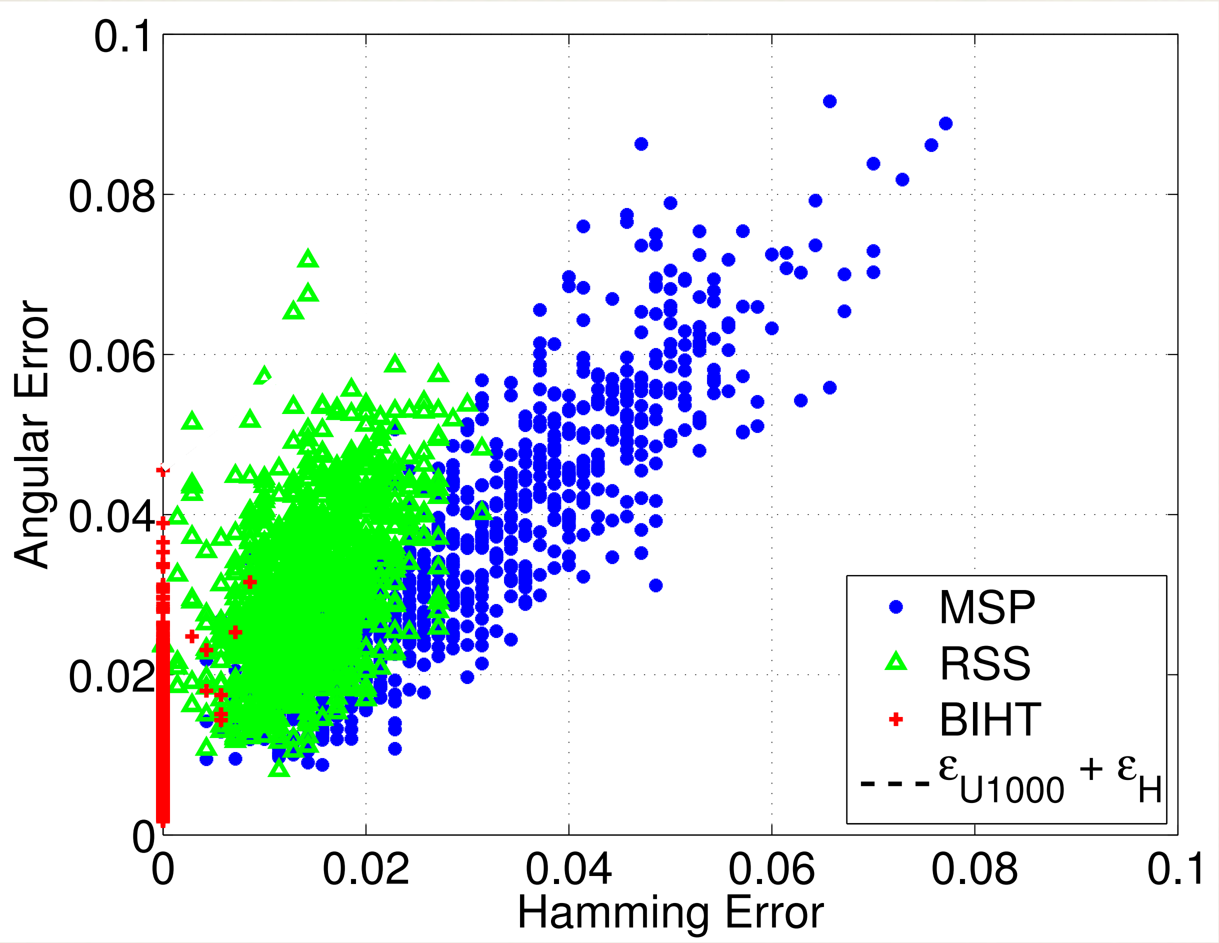
N = 1000, K = 10
Bernoulli-Gaussian model
normalized signals
1000 trials

Matching Sign pursuit (MSP)
Restricted-Step Shrinkage (RSS)
Binary Iterative Hard Thresholding (BIHT)

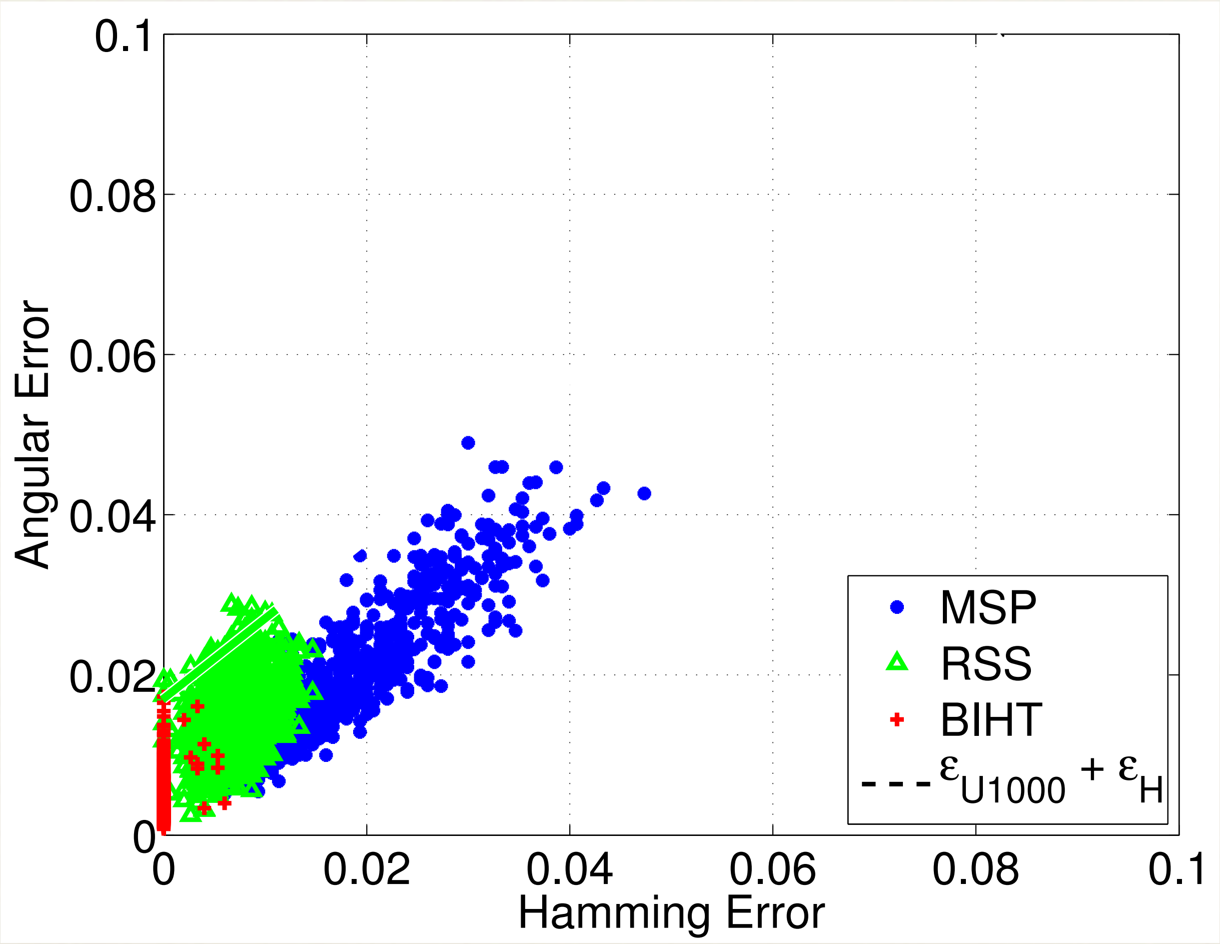
(tested also for signals sparse in DCT, wavelets, ...)

Simulations: Testing BϵSE

$$d_{\text{ang}}(\boldsymbol{x}, \boldsymbol{x}^*) - \epsilon(M) \leqslant d_H(A(\boldsymbol{x}), A(\boldsymbol{x}^*)) \leqslant d_{\text{ang}}(\boldsymbol{x}, \boldsymbol{x}^*) + \epsilon(M)$$



$M/N = 0.7$



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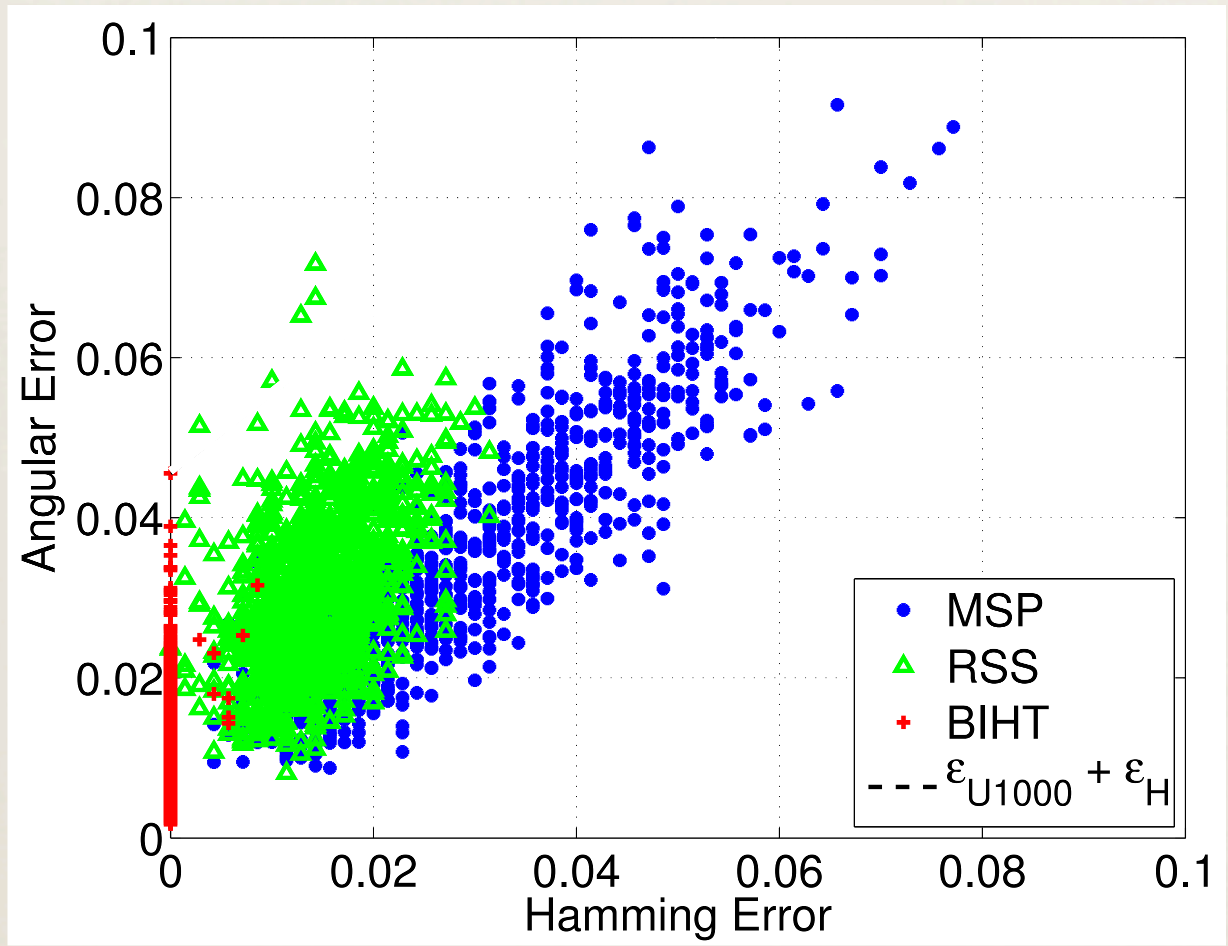
$$\underbrace{d_H(\boldsymbol{x}, \boldsymbol{x}^*)}_{C} - \epsilon(M) \leqslant \underbrace{d_{\mathrm{ang}}(A(\boldsymbol{x}), A(\boldsymbol{x}^*))}_{D} \leqslant \underbrace{d_H(\boldsymbol{x}, \boldsymbol{x}^*)}_{C} + \epsilon(M)$$

D



C

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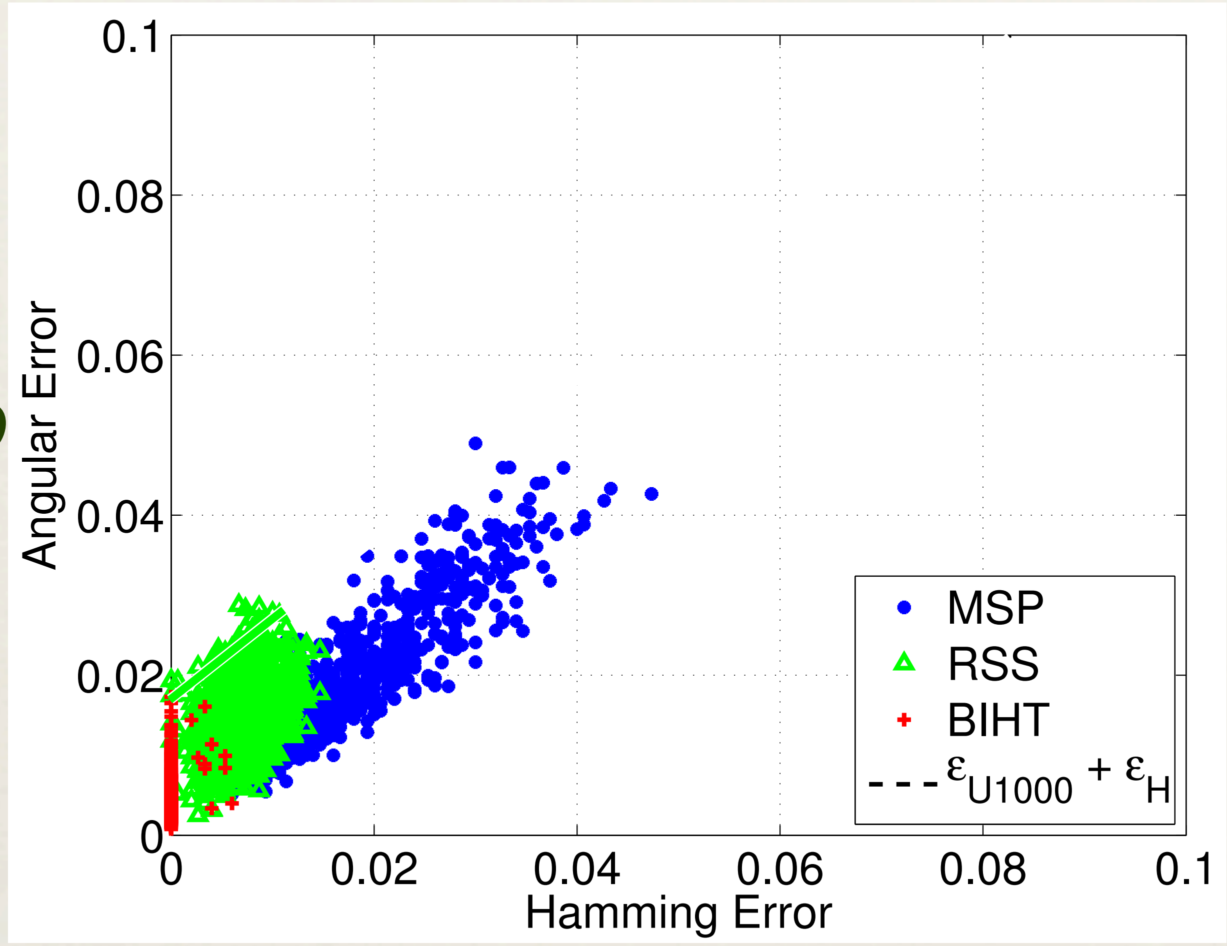

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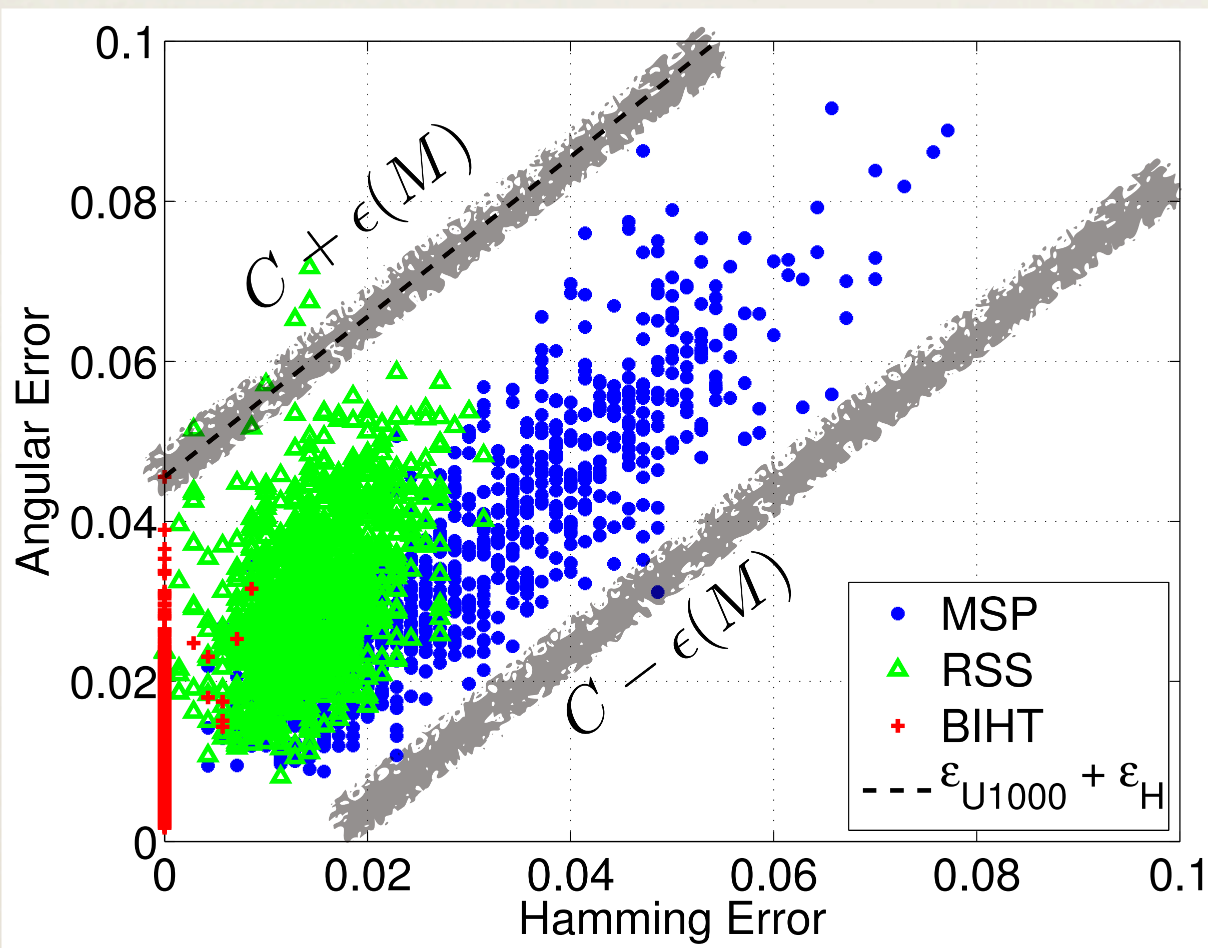

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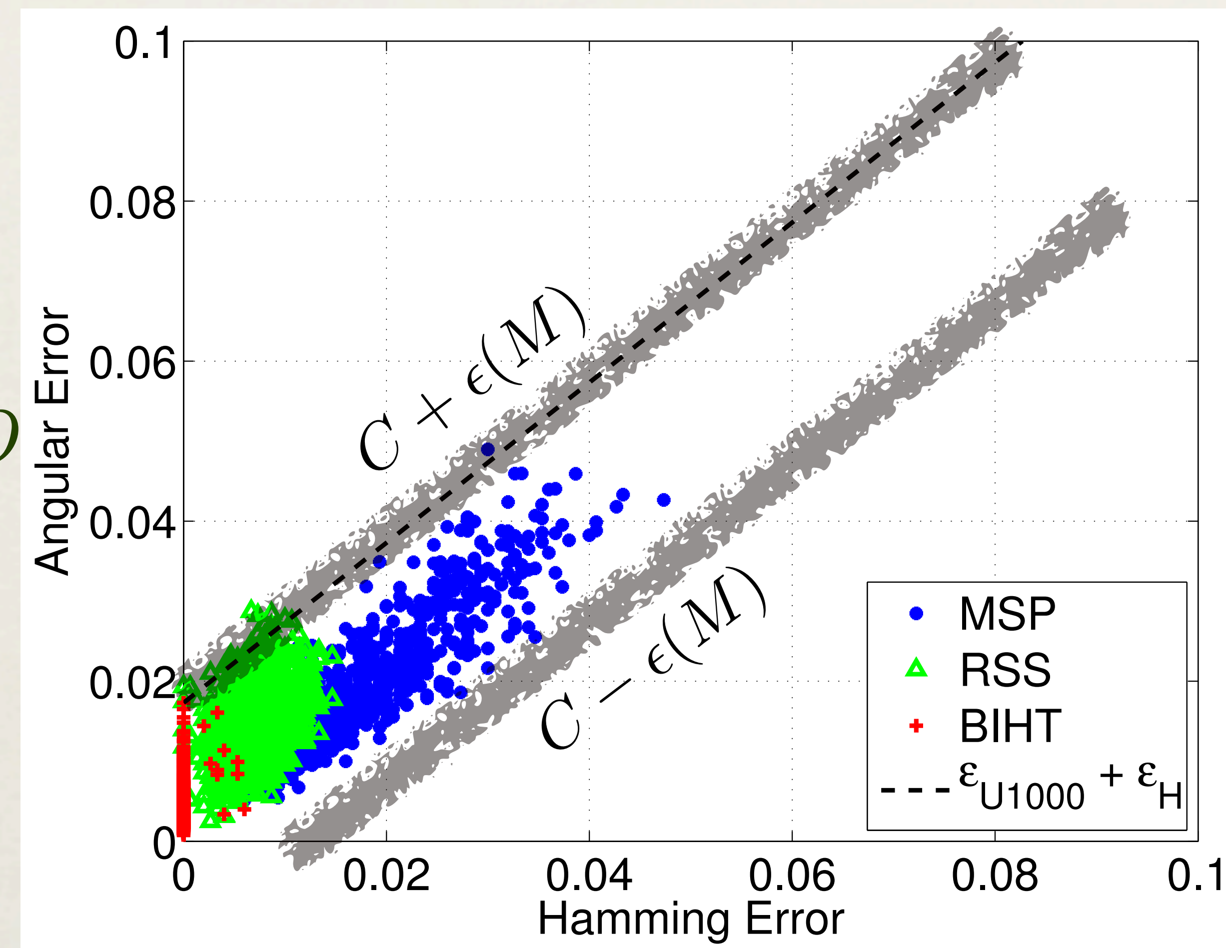

C

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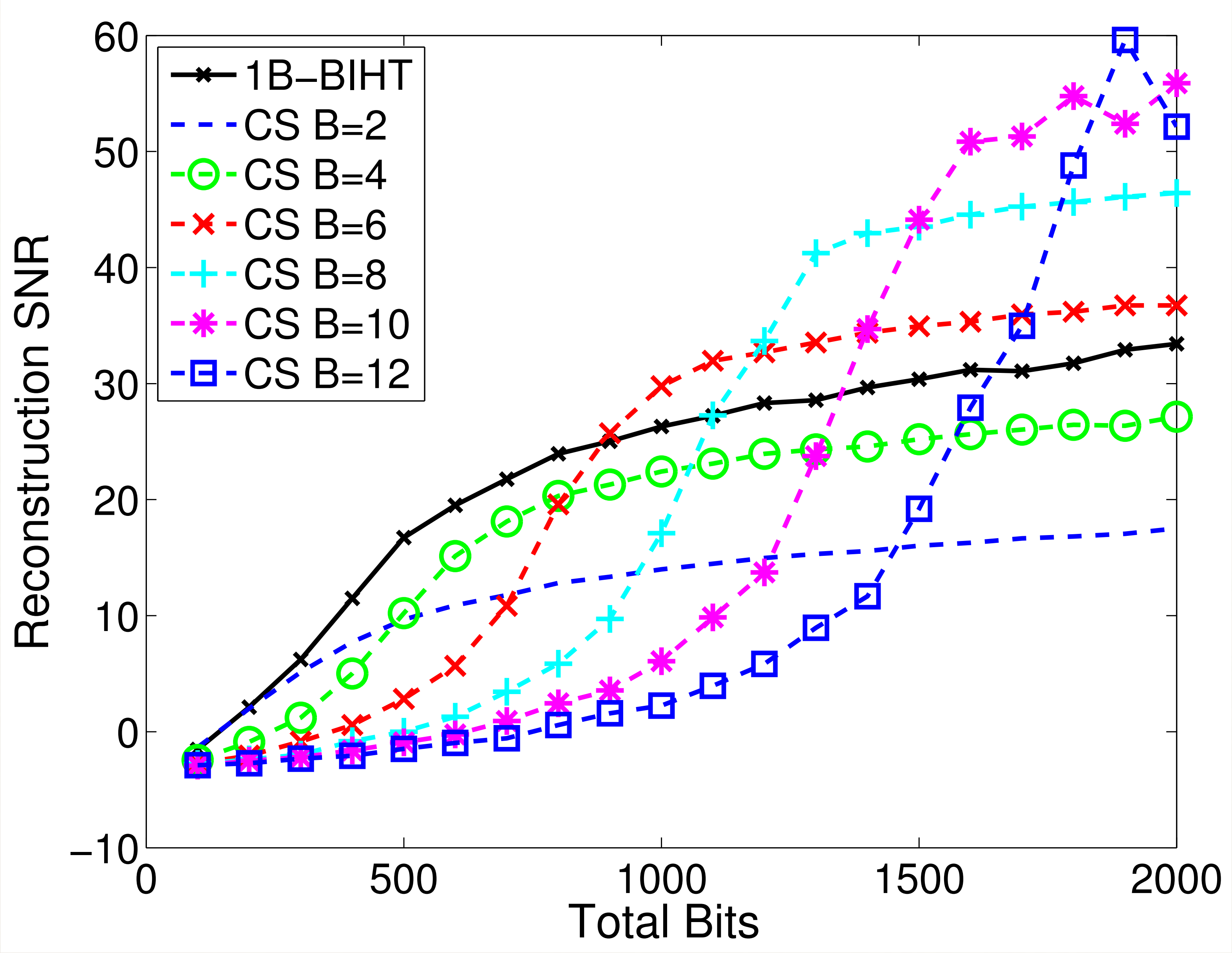
Final Comparison: CS vs bits/meas.



N = 2000, K = 20

Bernoulli-Gaussian model

normalized signals

B bits/measurement

B = 1, ..., 12

M = Total Bits/B

1000 trials

Conclusion & Discussion

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* Short BIHT matlab code available:
  http://perso.uclouvain.be/laurent.jacques/index.php/Main/BIHTDemo
* 1-Bit CS Resource Page: http://dsp.rice.edu/1bitCS

Subset of References (& Thank you for your attention)

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- **1-Bit CS Resource Page:** http://dsp.rice.edu/1bitCS/