Interferometric Single-pixel Imaging with a Multicore Fiber





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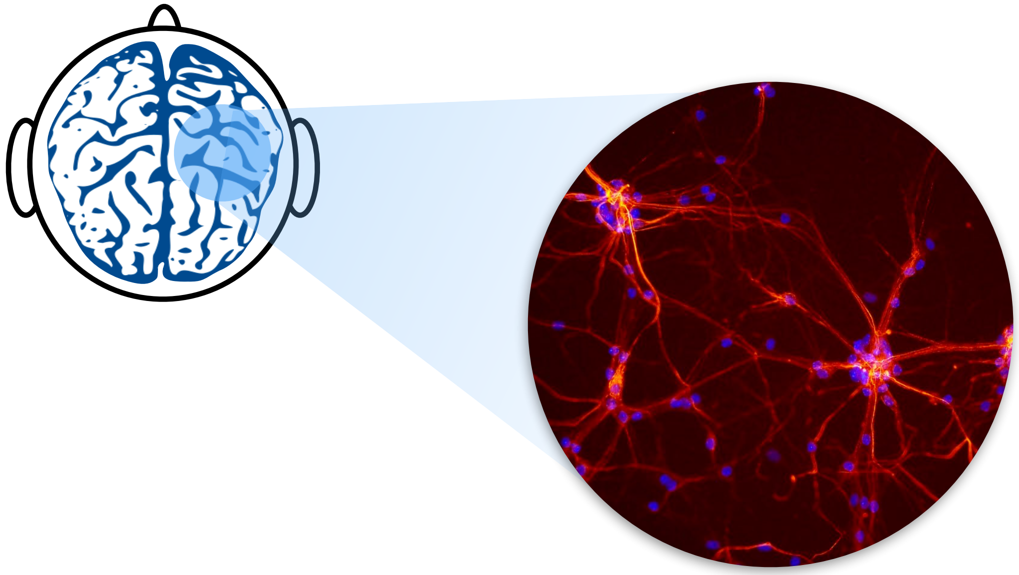
Seminar at CREATIS, Lyon, France

How to see neurons firing?



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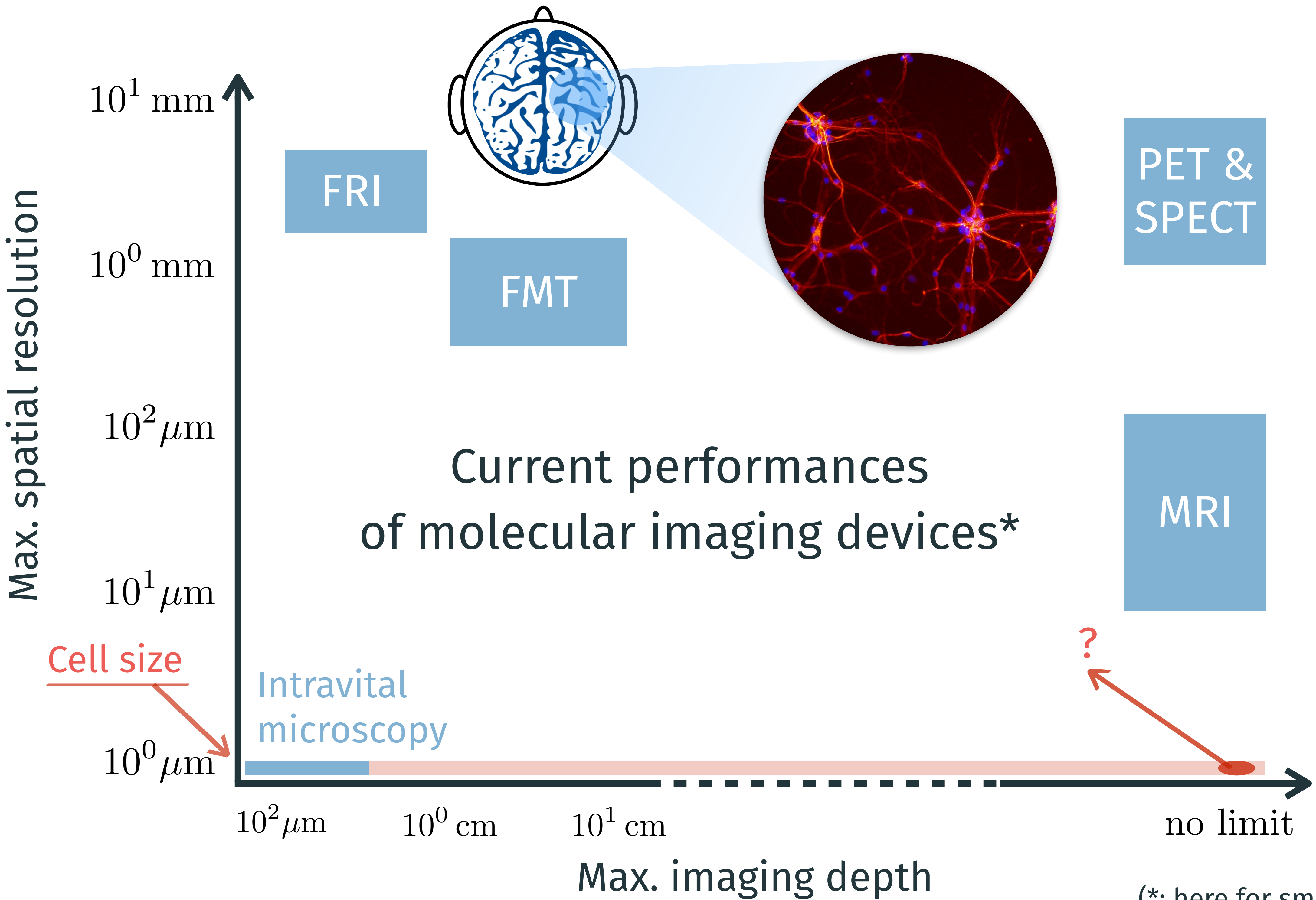

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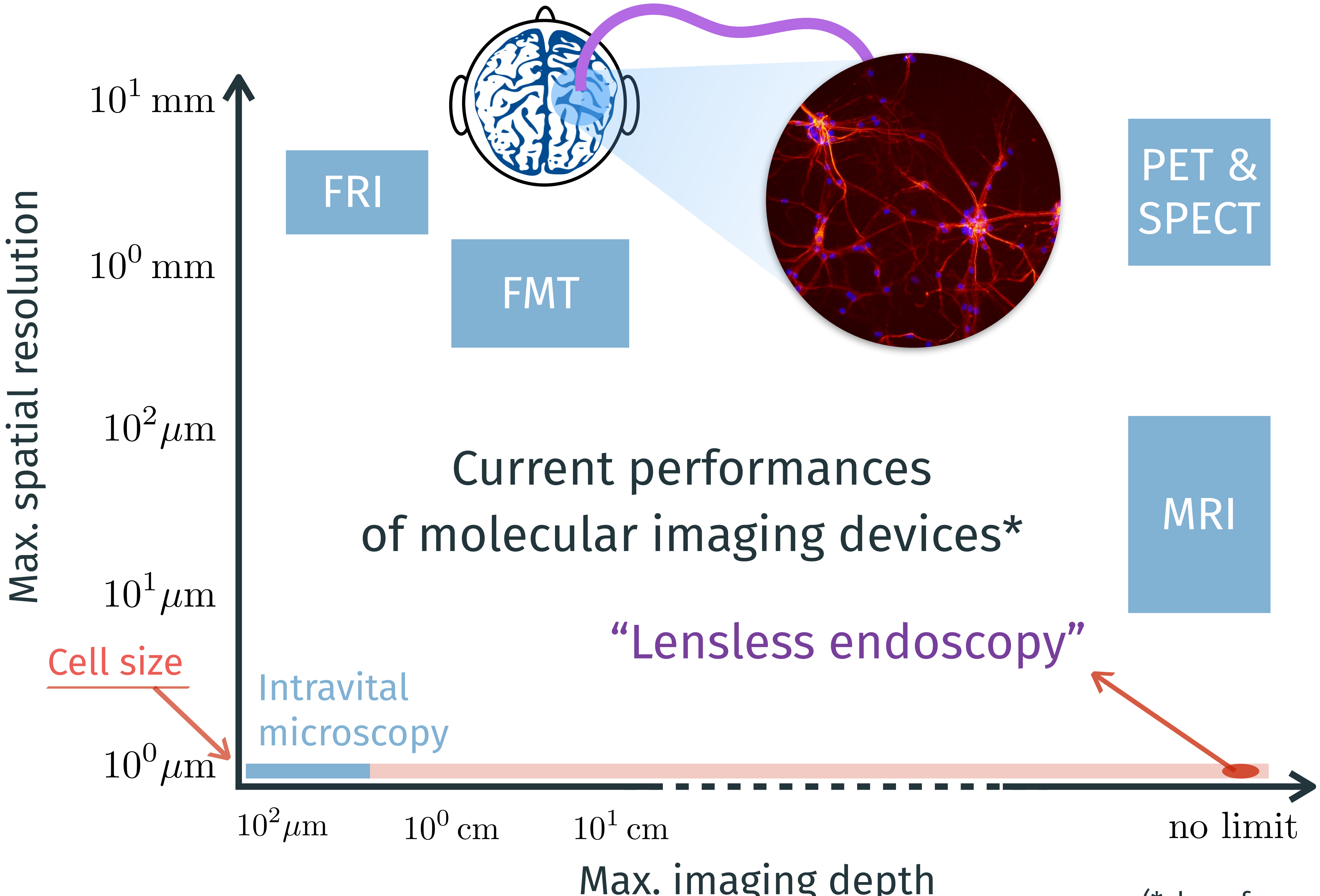

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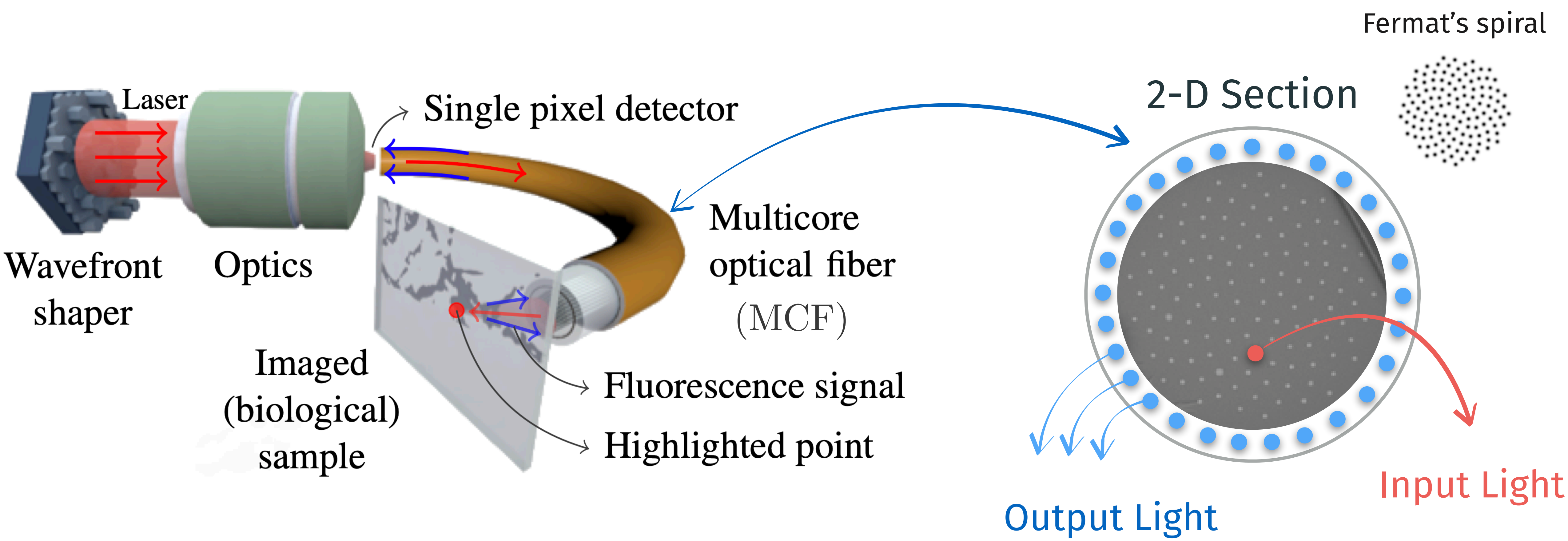
Lensless endoscopy: focused mode



E. R. Andresen, S. Sivankutty, V. Tsvirkun, et al., "Ultrathin endoscopes based on multicore fibers and adaptive optics: status and perspectives," Journal of Biomedical Optics, 2016.

S. Sivankutty, V. Tsvirkun, O. Vanvincq, et al., "Nonlinear imaging through a fermat's golden spiral multicore fiber," Optics letters, 2018.

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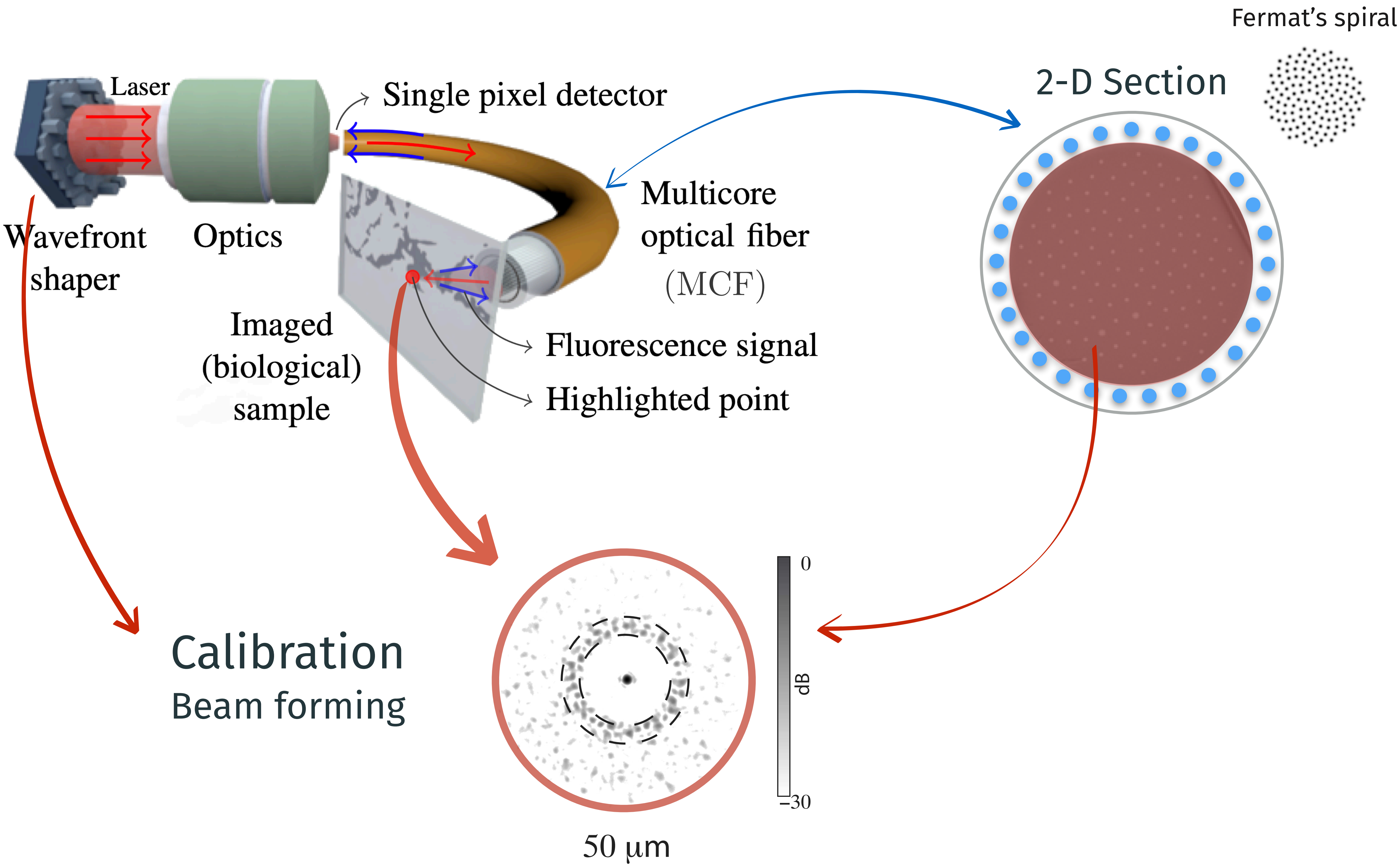

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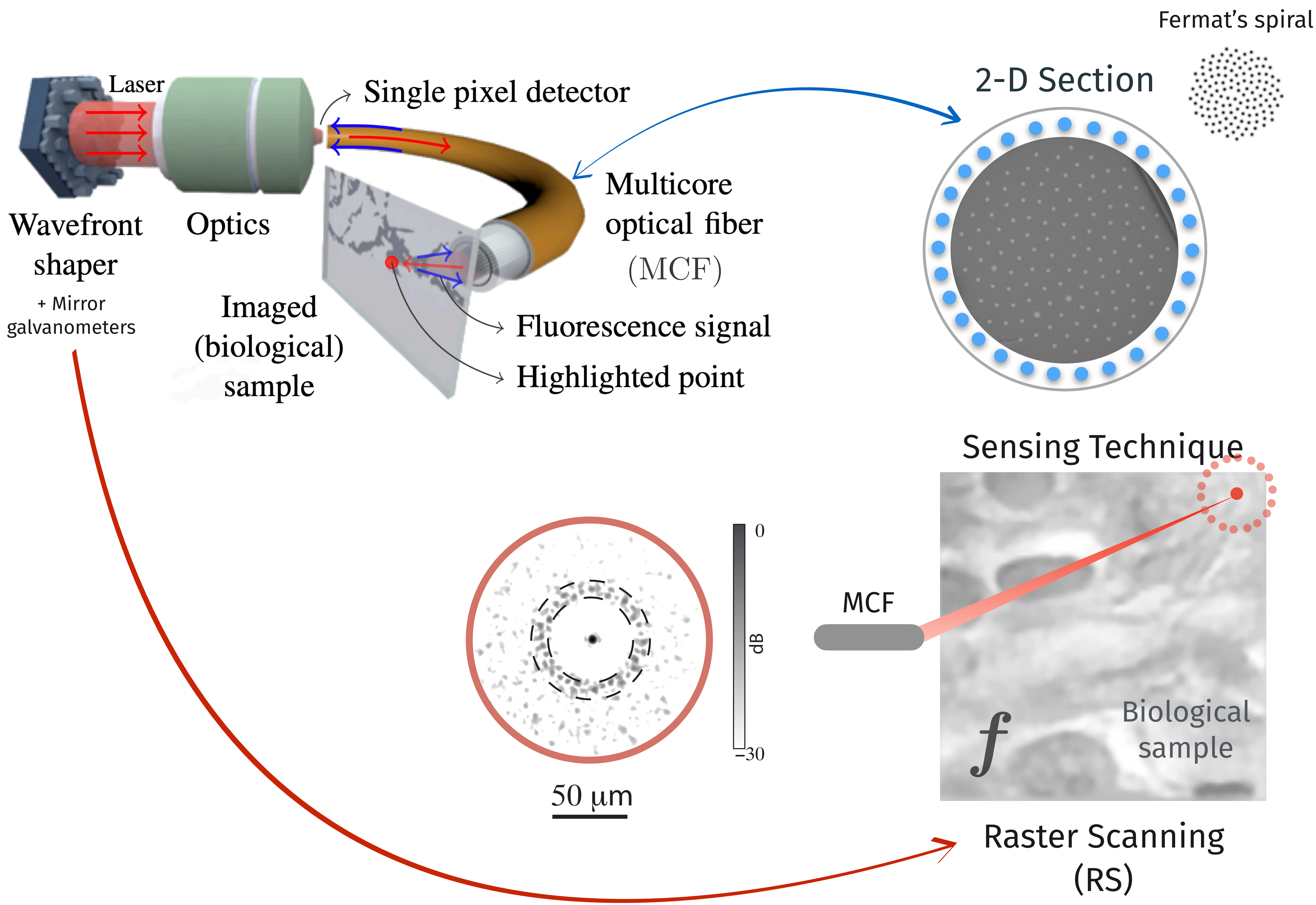

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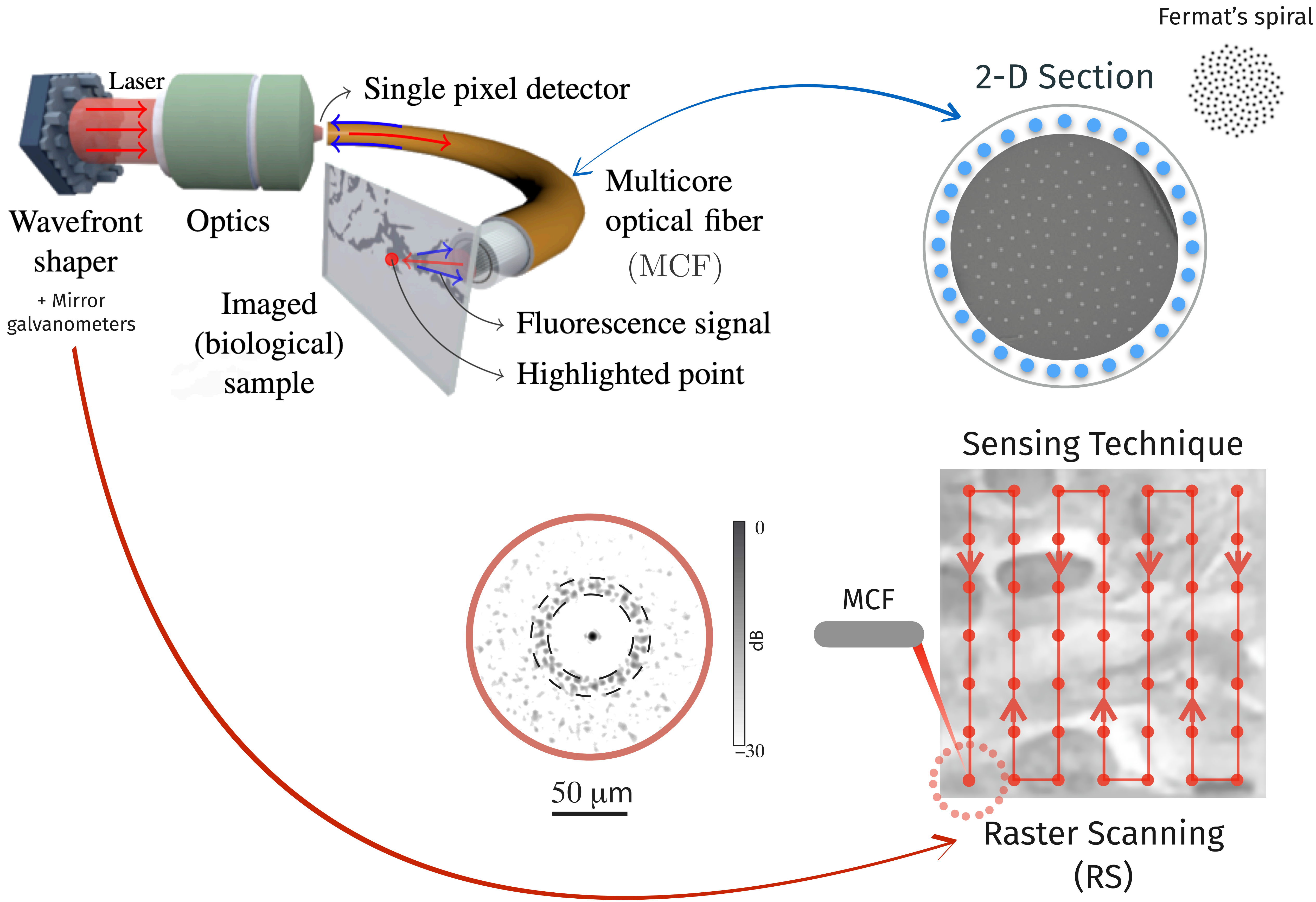

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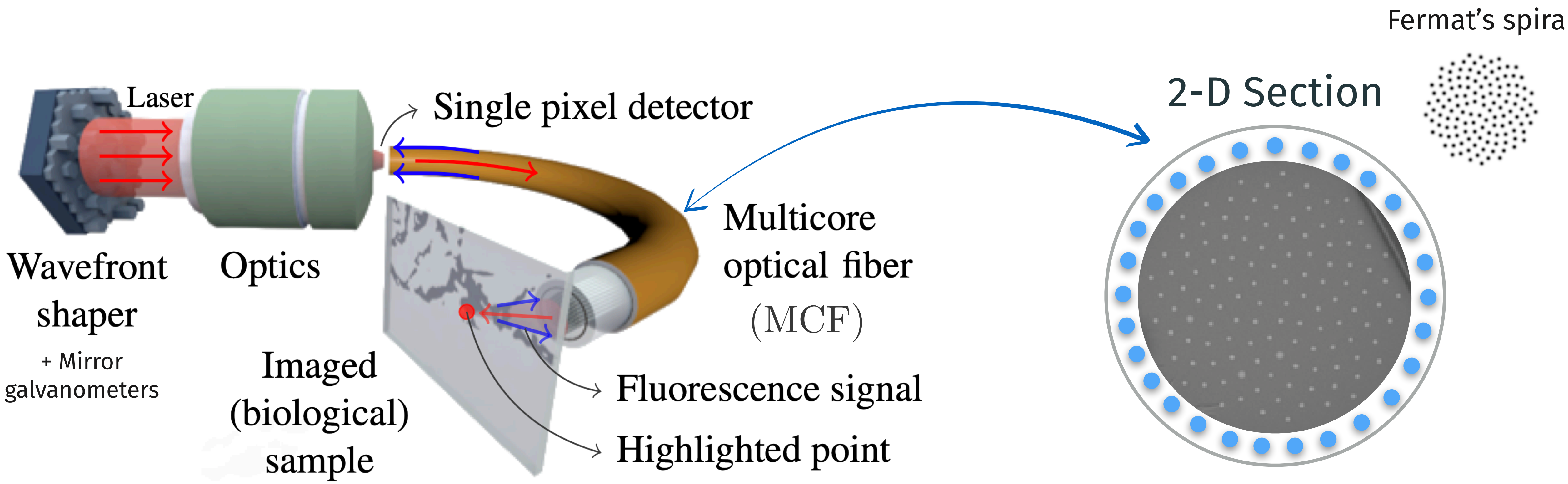
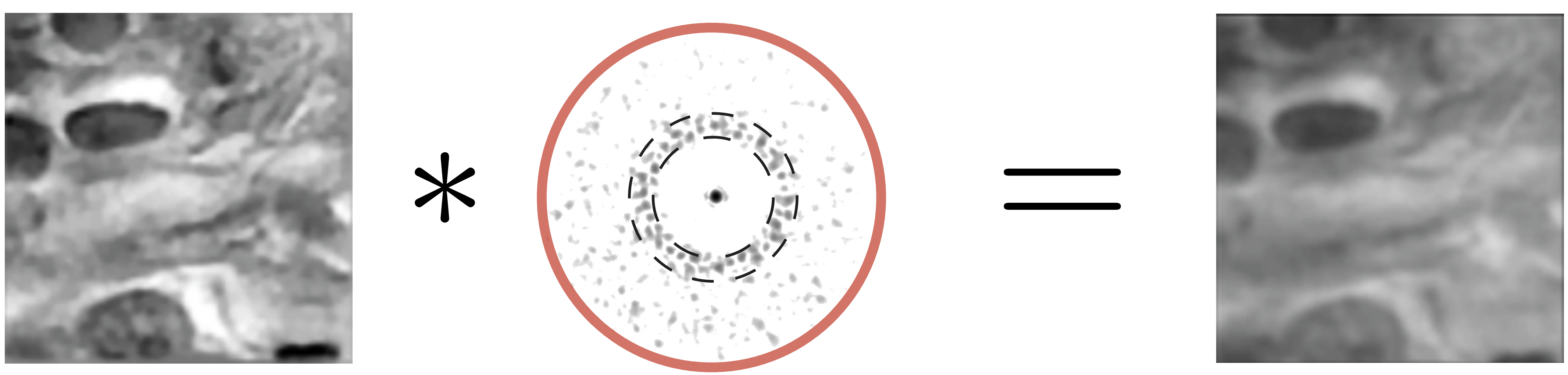


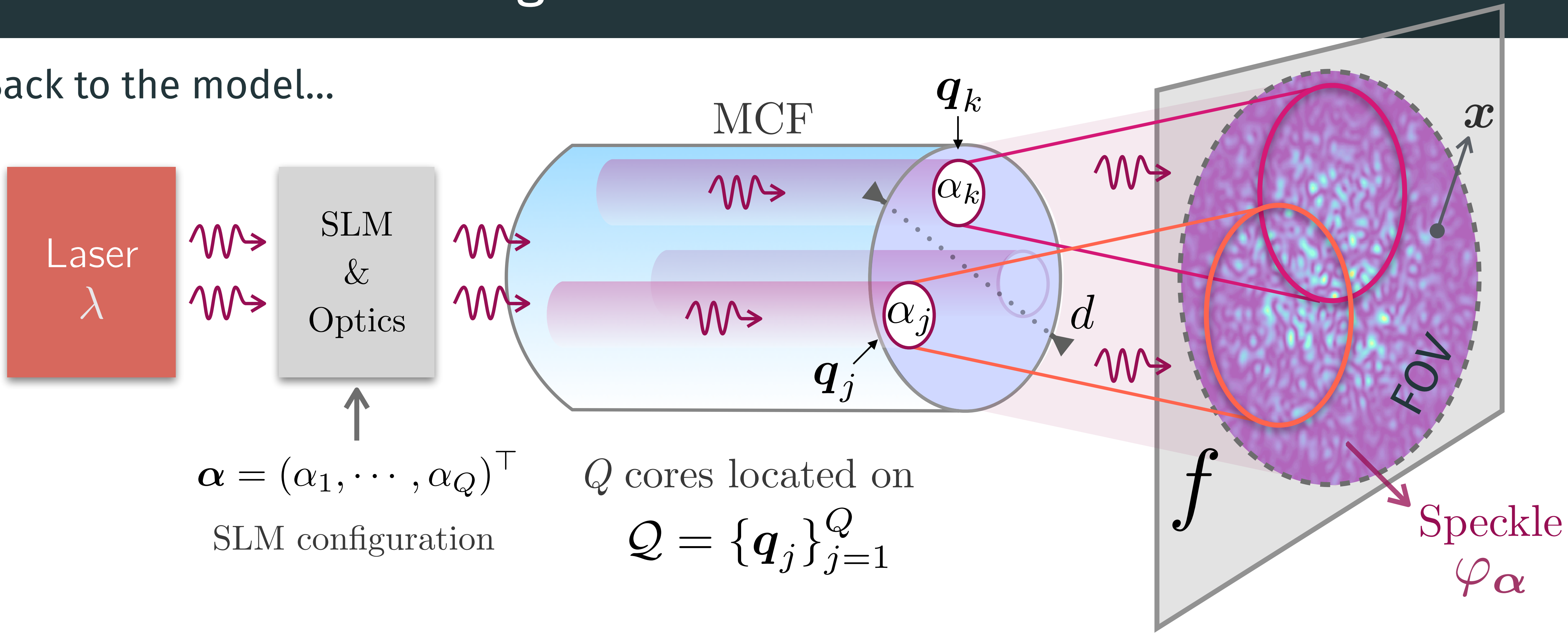


Sensing model

Direct Imaging

A closer look to sensing model

Back to the model...



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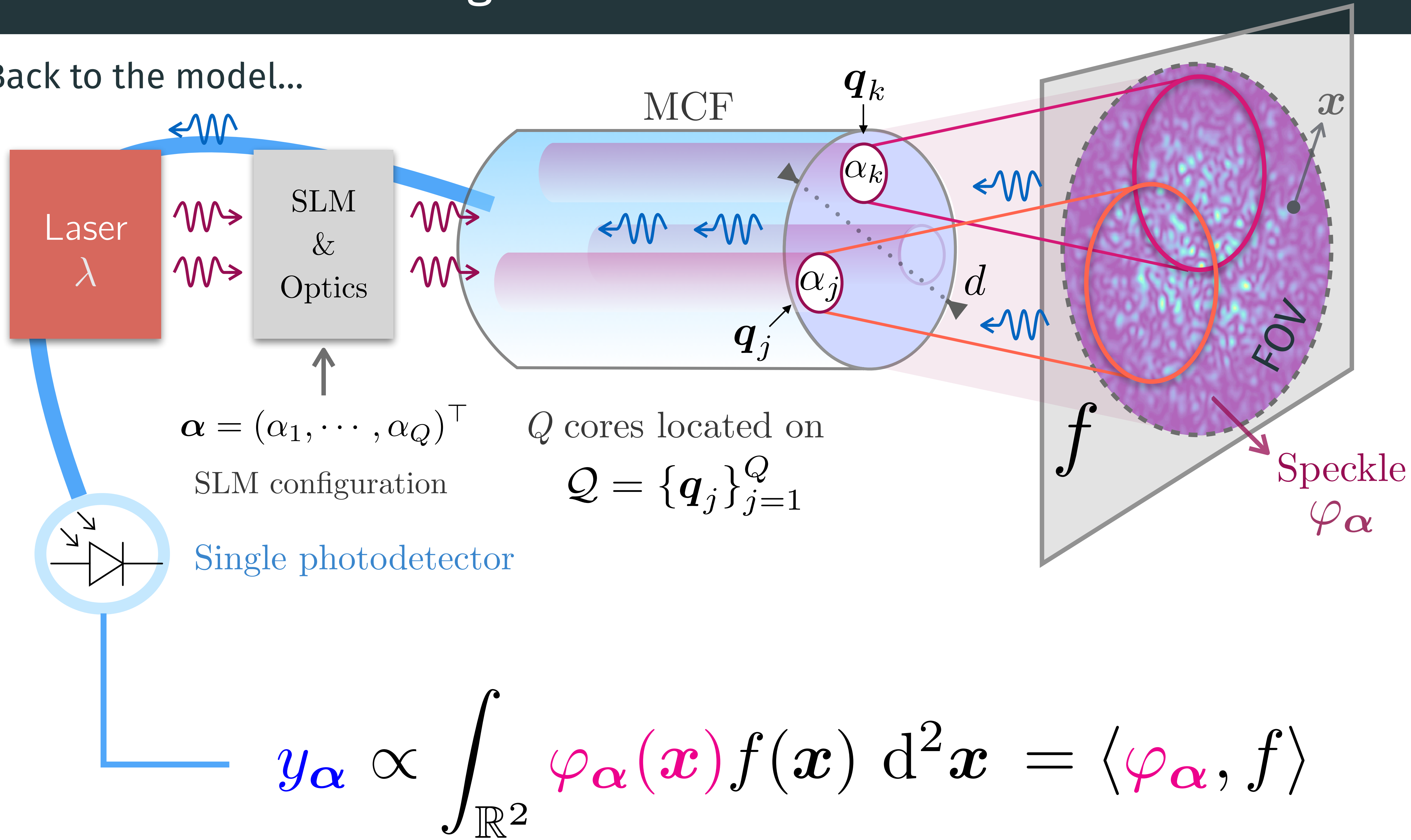
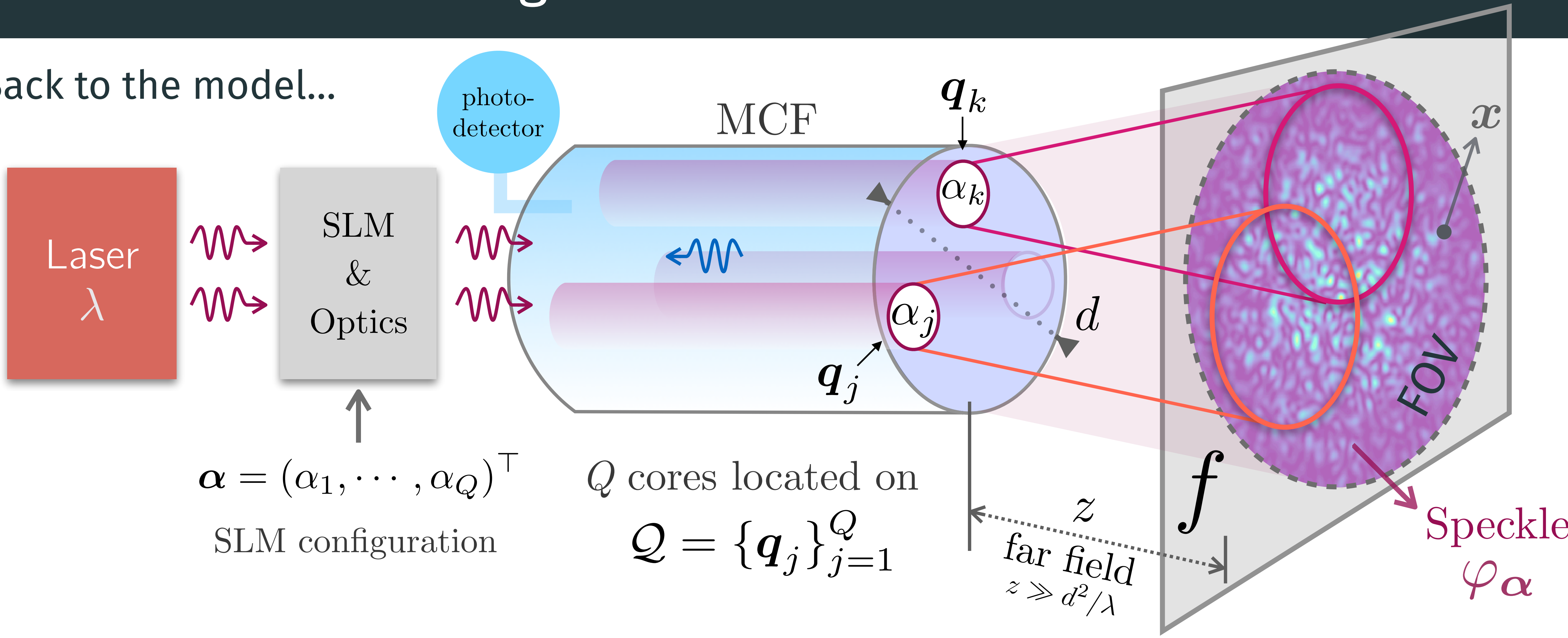


$$y_{\boldsymbol{\alpha}} \propto \int_{\mathbb{R}^2} \varphi_{\boldsymbol{\alpha}}(\boldsymbol{x}) f(\boldsymbol{x})\, \mathrm{d}^2\boldsymbol{x} = \langle \varphi_{\boldsymbol{\alpha}}, f \rangle$$

Measurement model

A closer look to sensing model

Back to the model...



Speckles are interferences: (Under far-field approximation)

$$\varphi_{\boldsymbol{\alpha}}(\boldsymbol{x}) \propto \underbrace{w(\boldsymbol{x})}_{\text{FOV window}} \sum_{j,k=1}^{Q} \alpha_j \alpha_k^* e^{\frac{2\pi \mathrm{i}}{\lambda z}(\boldsymbol{q}_j - \boldsymbol{q}_k)^\top \boldsymbol{x}}$$

Compressive Sensing? $\varphi_\alpha \equiv$ Random Gaussian pattern?

(noiseless) Interferometric sensing model

Given $\varphi_{\boldsymbol{\alpha}}(\boldsymbol{x}) = w(\boldsymbol{x}) \sum_{j,k=1}^{Q} \alpha_j \alpha_k^* e^{\frac{2\pi\mathrm{i}}{\lambda z}(\boldsymbol{q}_j - \boldsymbol{q}_k)^\top \boldsymbol{x}}$, we get

$$\langle \varphi_{\boldsymbol{\alpha}}, f \rangle = \sum_{j,k=1}^{Q} \alpha_j \alpha_k^* \left[\int_{\mathbb{R}^2} e^{\frac{2\pi\mathrm{i}}{\lambda z}(\boldsymbol{q}_j - \boldsymbol{q}_k)^\top \boldsymbol{x}} w(\boldsymbol{x}) f(\boldsymbol{x}) \mathrm{d}\boldsymbol{x} \right]$$

$$\rightarrow \boldsymbol{\alpha}^* \boldsymbol{\mathcal{I}}[wf]\, \boldsymbol{\alpha}$$

with the (Hermitian) *interferometric matrix* $\boldsymbol{\mathcal{I}}[wf] \in \mathbb{C}^{Q \times Q}$ s.t.

$$(\boldsymbol{\mathcal{I}}[wf])_{j,k} := \int_{\mathbb{R}^2} e^{\frac{2\pi\mathrm{i}}{\lambda z}(\boldsymbol{q}_j - \boldsymbol{q}_k)^\top \boldsymbol{x}} \, w(\boldsymbol{x}) f(\boldsymbol{x}) \, \mathrm{d}\boldsymbol{x}.$$

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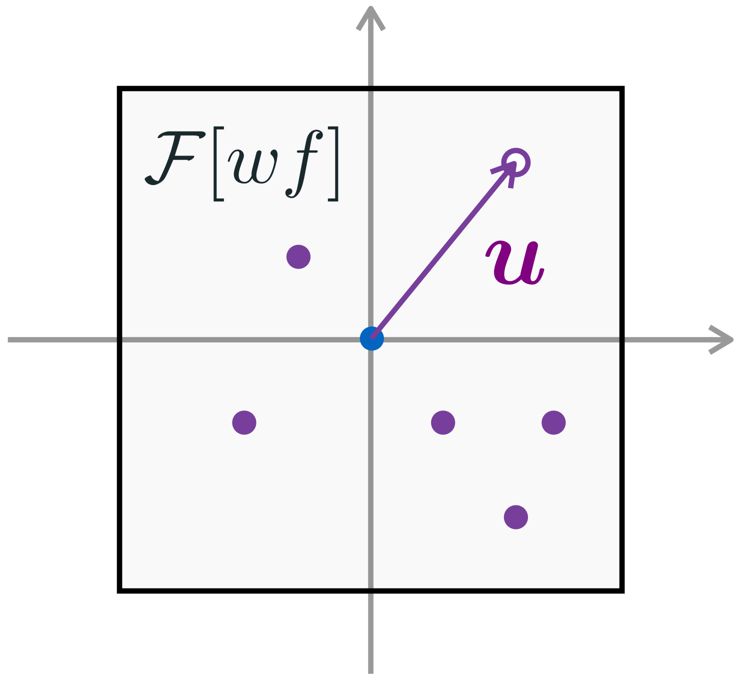

$$\boldsymbol{u} \in \mathcal{V} := \tfrac{1}{\lambda z}(\mathcal{Q} - \mathcal{Q})$$

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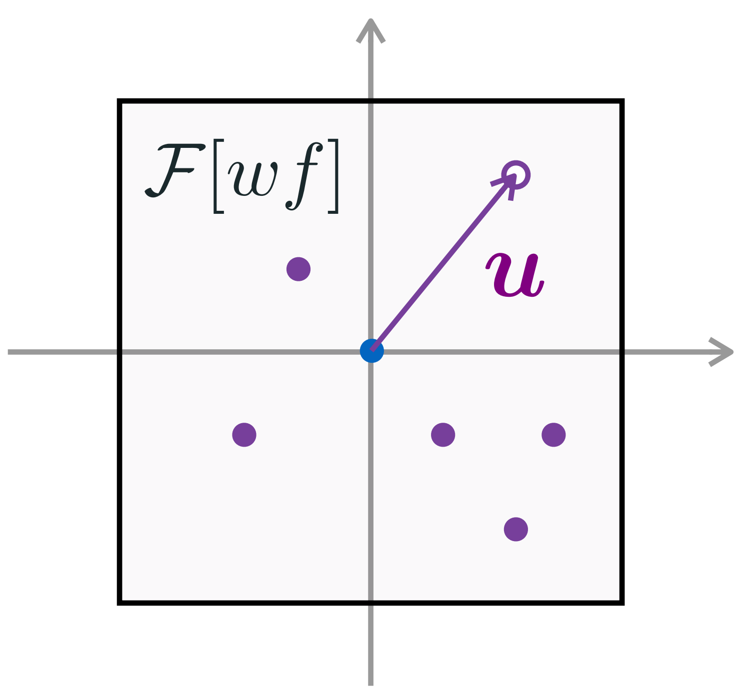
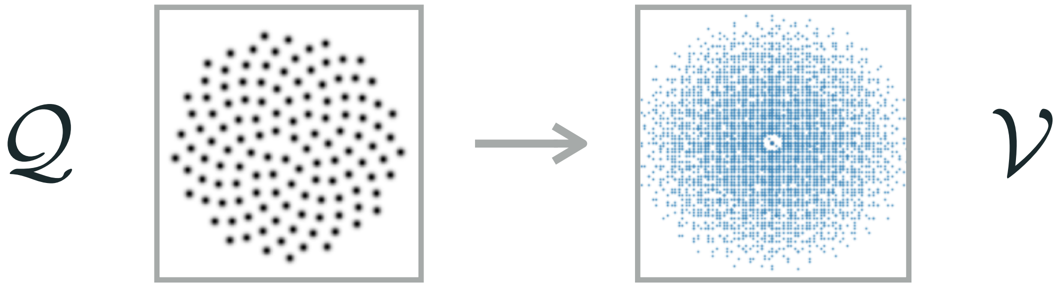


$$\boldsymbol{u} \in \mathcal{V} := \tfrac{1}{\lambda z}(\mathcal{Q} - \mathcal{Q})$$

Observation 1: denser Fourier sampling if

$$|\mathcal{V}| \simeq Q^2$$

- Lattices are bad core arrangements
- Fermat's spiral is not bad



(noiseless) Interferometric sensing model

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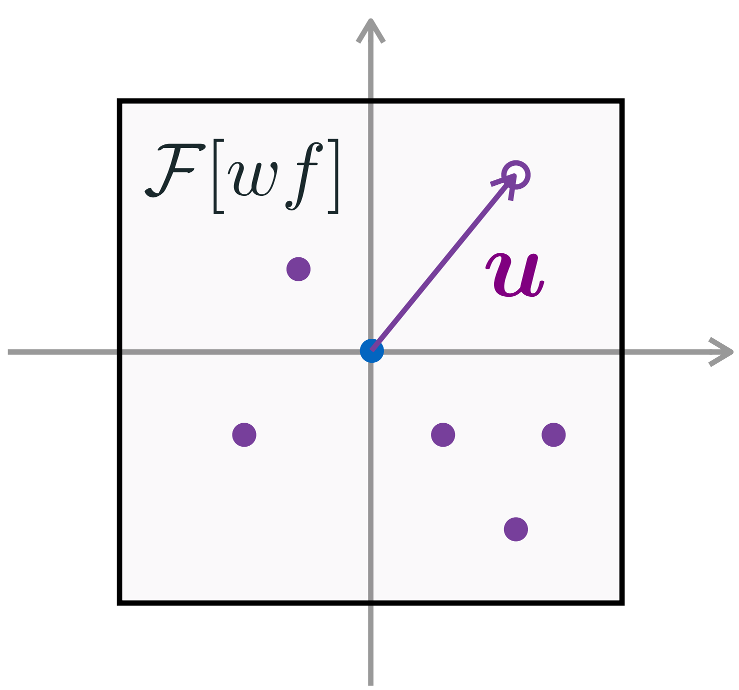

$\boldsymbol{u} \in \mathcal{V} := \frac{1}{\lambda z}(\mathcal{Q} - \mathcal{Q})$

Observation 2:

Low-complexity on f
→
Low-complexity on $\boldsymbol{\mathcal{I}}$.

e.g., sparsity → low-rank

(noiseless) Interferometric sensing model

Given $\varphi_{\boldsymbol{\alpha}}(\boldsymbol{x}) = w(\boldsymbol{x}) \sum_{j,k=1}^{Q} \alpha_j \alpha_k^* e^{\frac{2\pi i}{\lambda z}(\boldsymbol{q}_j - \boldsymbol{q}_k)^\top \boldsymbol{x}}$, we get

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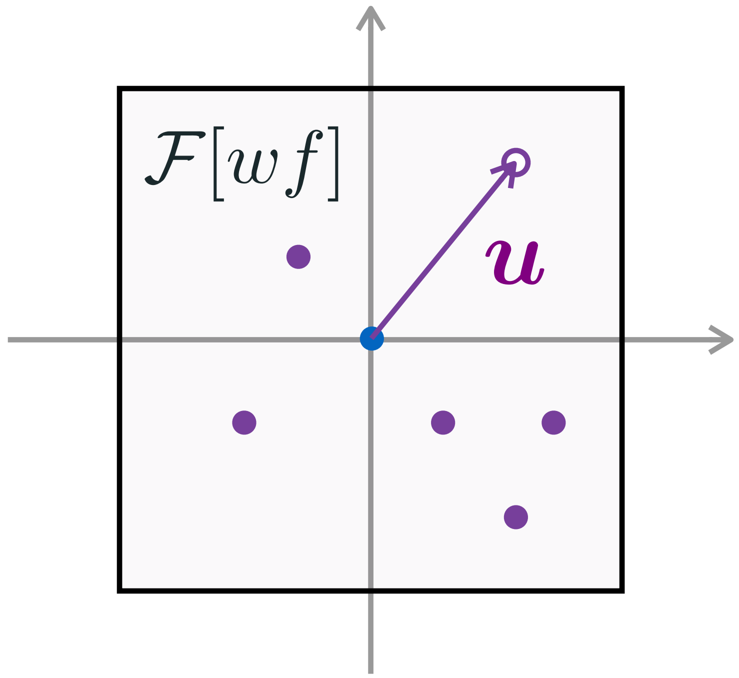

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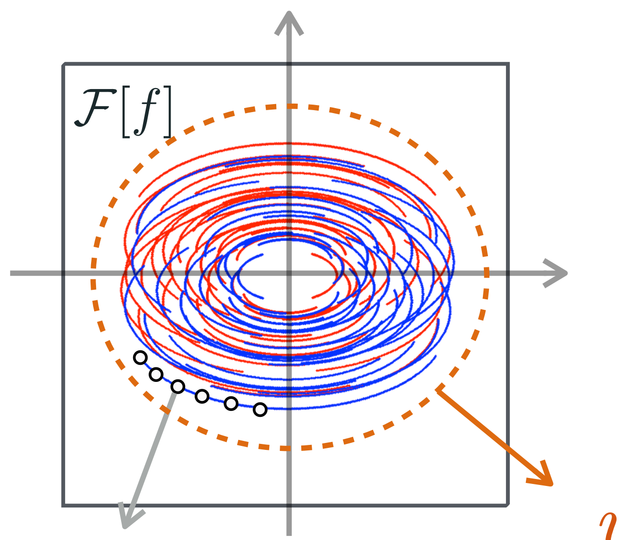
Observation 3: Similarity with radioastronomy!



Noisy model:

$$y_{\boldsymbol{\alpha}} = \boldsymbol{\alpha}^* \boldsymbol{\mathcal{I}}[wf]\,\boldsymbol{\alpha} + \text{noise} = \underbrace{\langle \boldsymbol{\alpha}\,\boldsymbol{\alpha}^*, \boldsymbol{\mathcal{I}}[wf]\rangle_{\mathrm{F}}}_{\substack{\text{Rank-one projection}\\ \text{of } \boldsymbol{\mathcal{I}}[wf]}} + \text{noise},$$

(or ROP)

General model: Over m SLMs configs $\{\boldsymbol{\alpha}_j\}_{j=1}^m$, we get

$$\boldsymbol{y} = (y_{\boldsymbol{\alpha}_1}, \cdots, y_{\boldsymbol{\alpha}_m})^\top = \mathcal{A}(\boldsymbol{\mathcal{I}}[wf]) + \text{noise},$$

with the ROP operator: $\mathcal{A}(\boldsymbol{M}) := \{\langle \boldsymbol{\alpha}_j \boldsymbol{\alpha}_j^*, \boldsymbol{M}\rangle_{\mathrm{F}}\}_{j=1}^m$.

Chen, Y., Chi, Y., & Goldsmith, A. J. (2015). Exact and stable covariance estimation from quadratic sampling via convex programming. *IEEE Transactions on Information Theory*, *61*(7), 4034-4059.

Cai, T. T., & Zhang, A. (2015). ROP: Matrix recovery via rank-one projections. The Annals of Statistics, 43(1), 102-138.

Composition of two sensing methods

$$\boldsymbol{y} = (y_{\boldsymbol{\alpha}_1}, \cdots, y_{\boldsymbol{\alpha}_m})^\top = \underbrace{\mathcal{A}}_{②\ m \times Q^2}(\overbrace{\mathcal{I}}^{Q \times Q\ ①}[wf]) + \text{noise},$$

Sample complexities of interest:

② Does $\mathcal{A}$ capture enough from $\mathcal{I}$? $\leftrightarrow$ m big enough?

① Does $\mathcal{I}$ capture enough from f? $\leftrightarrow$ Q big enough?

Core arrangement?

Composition of two sensing methods

$Q \times Q$ (1)

$$\boldsymbol{y} = (y_{\boldsymbol{\alpha}_1}, \cdots, y_{\boldsymbol{\alpha}_m})^\top = \mathcal{A}(\mathcal{I}[wf]) + \text{noise},$$

(2) $m \times Q^2$

Sample complexities of interest:

(2) Does $\mathcal{A}$ capture enough from $\mathcal{I}$? $\leftrightarrow$ m big enough?

(1) Does $\mathcal{I}$ capture enough from f? $\leftrightarrow$ Q big enough?

Core arrangement?

A few answers from a few simplifications ...

Theory + Simulations + Experimental results

Theoretical guarantees: Assumptions (6)

H1 **Bounded FOV** : $\operatorname{supp} w \subset [-\frac{L}{2}, \frac{L}{2}] \times [-\frac{L}{2}, \frac{L}{2}]$

H2 **Bandlimited f** :

$\rightarrow$ (H1 & H2) $\boldsymbol{f} \in \mathbb{R}^N$ = sampling of $w(x)f(x)$ over a N pixel grid.

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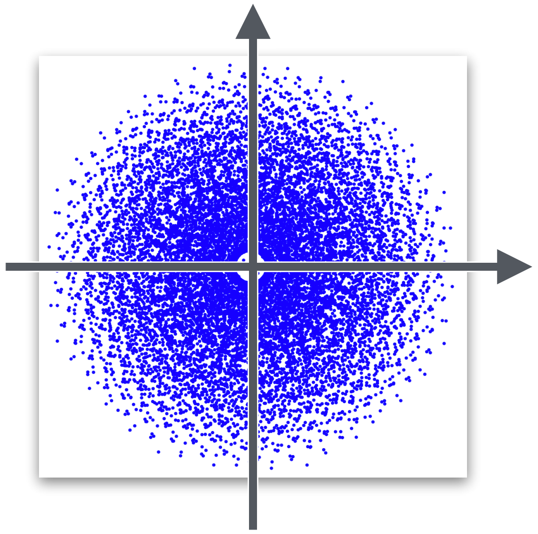
H3 K-sparse $\boldsymbol{f} \in \mathbb{R}^N$ ($\equiv$ pixel basis)



H4 Distinct, on-grid, non-zero visibilities

$$|\mathcal{V}_0| \simeq Q^2, \text{ with } \mathcal{V}_0 = \mathcal{V} \setminus \{\mathbf{0}\}$$

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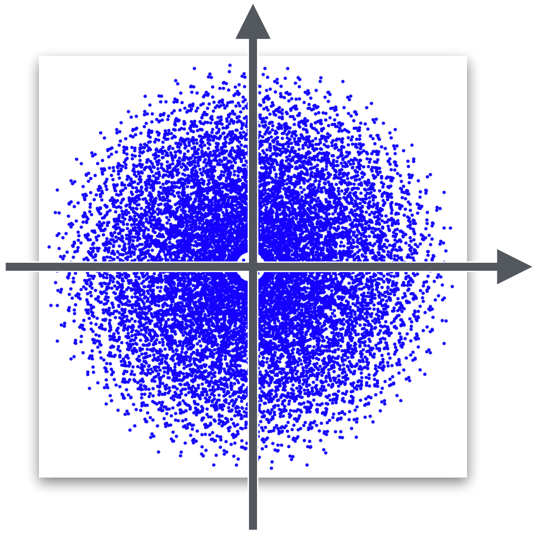
H3 **K-sparse $f \in \mathbb{R}^N$** (≡ pixel basis)



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$$|\mathcal{V}_0| \simeq Q^2, \text{ with } \mathcal{V}_0 = \mathcal{V} \setminus \{\mathbf{0}\}$$

H5 **RIP Fourier Sensing**: For $\boldsymbol{\Phi} \equiv$ partial random Fourier sampling on $\mathcal{V}_0$,

$$\boldsymbol{\Phi} \text{ is RIP}(K, \delta) \text{ if } |\mathcal{V}_0| \gtrsim \delta^{-2} K \,\text{plog}(N, K, \delta),$$

$$i.e.\ \|\boldsymbol{\Phi u}\|^2 \simeq_\delta \|\boldsymbol{u}\|^2, \forall\ K\text{-sparse } \boldsymbol{u} \in \mathbb{R}^N$$

H6 **Unit module *sketching vector***: $\boldsymbol{\alpha}_j \sim_{\text{iid}} \boldsymbol{\alpha}_0$, with $|\alpha_k| = 1$.

Theoretical guarantees: Implications

Under previous assumptions:

For $j, k \in [Q]$, $j \neq k$, up to a reshaping $\mathcal{R}$,

$$(\boldsymbol{\mathcal{I}}[wf])_{jj} = (\boldsymbol{\mathcal{I}}[\boldsymbol{f}])_{jj} = \frac{1}{N}(\textstyle\sum_j f_j) \qquad \text{(DC part)}$$

$$(\boldsymbol{\mathcal{I}}[wf])_{jk} = (\boldsymbol{\mathcal{I}}[\boldsymbol{f}])_{jk} = (\underbrace{\mathcal{R}(}_{\text{reshaping as a matrix}}\boldsymbol{\Phi f}))_{jk} \qquad \text{(AC part)}$$

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$$(\boldsymbol{\mathcal{I}}[w\boldsymbol{f}])_{jk} = (\boldsymbol{\mathcal{I}}[\boldsymbol{f}])_{jk} = (\mathcal{R}(\boldsymbol{\Phi}\boldsymbol{f}))_{jk} \qquad \text{(AC part)}$$

Stabilisation of the ROP operator → *debiasing*

$$\boldsymbol{\mathcal{A}}^{\mathrm{c}} : \boldsymbol{\mathcal{J}} \in \mathcal{H}^Q \mapsto \big(\langle \boldsymbol{A}^{\mathrm{c}}_m, \boldsymbol{\mathcal{J}}\rangle\big)_{m=1}^{M} \in \mathbb{R}^M,$$

with $\boldsymbol{A}^{\mathrm{c}}_m := \boldsymbol{\alpha}_m \boldsymbol{\alpha}_m^* - \frac{1}{M}\sum_{j=1}^{M} \boldsymbol{\alpha}_j \boldsymbol{\alpha}_j^*$

Equivalence: Given $\boldsymbol{y} = \boldsymbol{\mathcal{A}}(\boldsymbol{\mathcal{J}})$,

$$\underbrace{y^{\mathrm{c}}_k := y_k - \tfrac{1}{M}\textstyle\sum_{j=1}^{M} y_j}_{\text{centering}} \text{ for } k \in [M] \Rightarrow \boldsymbol{y}^{\mathrm{c}} = \boldsymbol{\mathcal{A}}^{\mathrm{c}}(\boldsymbol{\mathcal{J}}).$$

Theoretical guarantees: Proposed reconstruction

$$\tilde{\boldsymbol{f}} = \arg\min_{\boldsymbol{v}\in\mathbb{R}^N} \|\boldsymbol{v}\|_1 \text{ s.t. } \Big\|\boldsymbol{y}^{\mathrm{c}} - \underbrace{\varpi\, \boldsymbol{\mathcal{A}}^{\mathrm{c}}\big(\mathcal{R}(\boldsymbol{\Phi}\boldsymbol{v})\big)}_{=:\boldsymbol{\mathcal{B}}(\boldsymbol{v})}\Big\|_1 \leq \epsilon$$

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$\boldsymbol{\mathcal{B}}$ has the $\mathrm{RIP}_{\ell_2/\ell_1}(K, \mathsf{m}_K, \mathsf{M}_K)$ w.h.p:

Under **H1-H6**, if $M \geq CK\ln(\frac{12eN}{K})$ and $Q(Q-1) \geq 4K\,\mathrm{plog}(N, K, \delta)$, $\exists 0 < \mathsf{m}_K < \mathsf{M}_K$ such that, w.h.p,

$$\mathsf{m}_K\|\boldsymbol{v}\| \leq \tfrac{1}{M}\|\boldsymbol{\mathcal{B}}(\boldsymbol{v})\|_1 \leq \mathsf{M}_K\|\boldsymbol{v}\|, \quad \forall \boldsymbol{v} \in \Sigma_K.$$

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Instance optimality:

Provided $\boldsymbol{\mathcal{B}}$ has the $\mathrm{RIP}_{\ell_2/\ell_1}(K', \mathsf{m}'_K, \mathsf{M}'_K)$, for $K' = O(K)$, $\exists C_0, D_0 > 0$,

$$\|\boldsymbol{f} - \tilde{\boldsymbol{f}}\| \le C_0\frac{\|\boldsymbol{f}-\boldsymbol{f}_K\|_1}{\sqrt{K}} + D_0\frac{\epsilon}{M}.$$

1-D simulations: phase transition diagrams

Simplified setting:

- 1-D core arrangement, $N = 256$
- K-sparse vectors
- Random $\{\boldsymbol{\alpha}_j\}_{j=1}^{M}$
- Q, M, K varying
- 80 trials, Success if $\geq$ 40 dB

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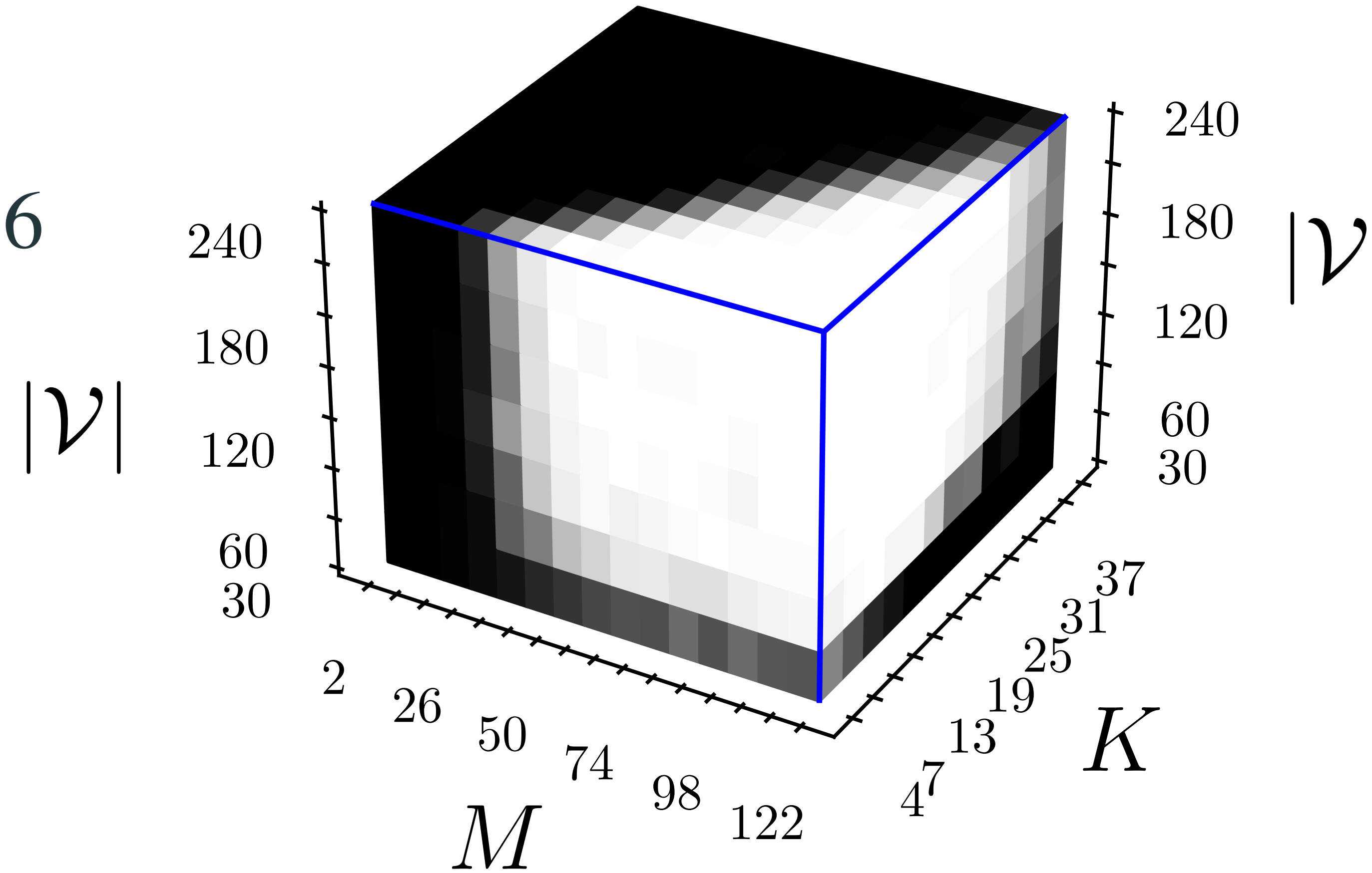
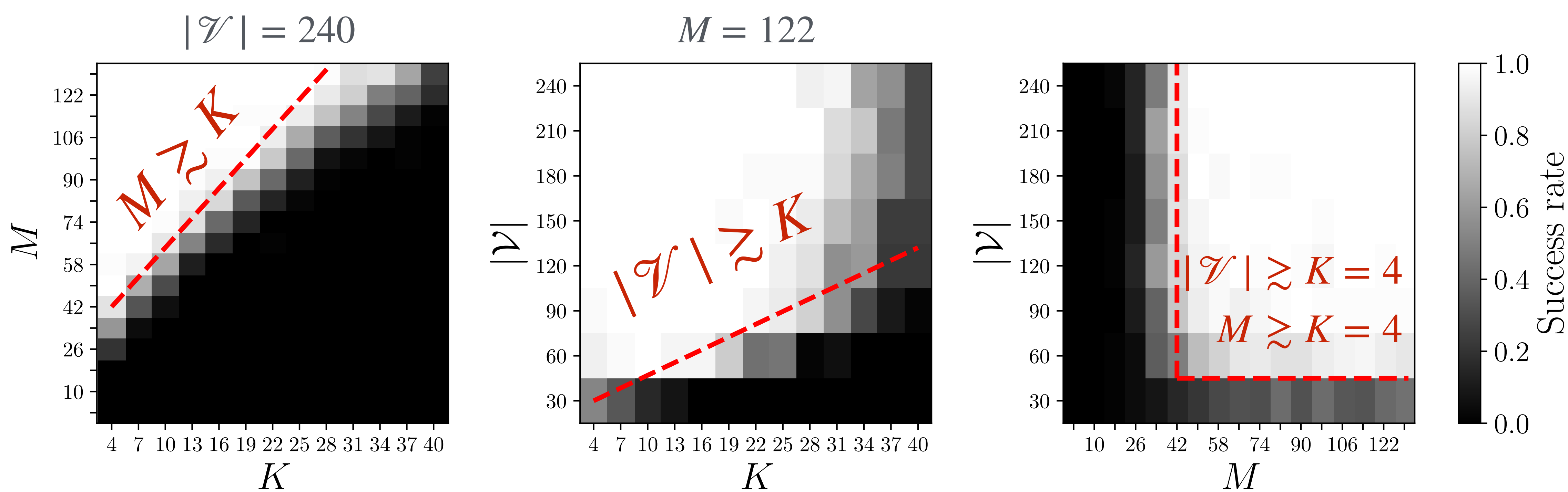
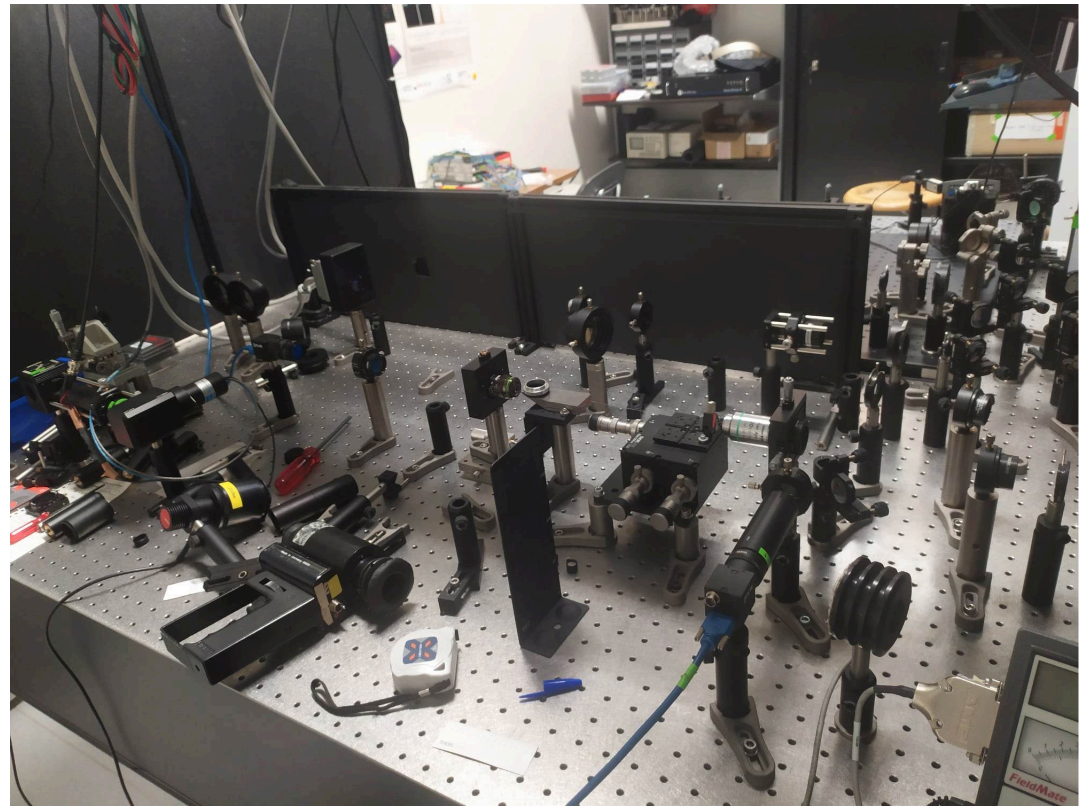
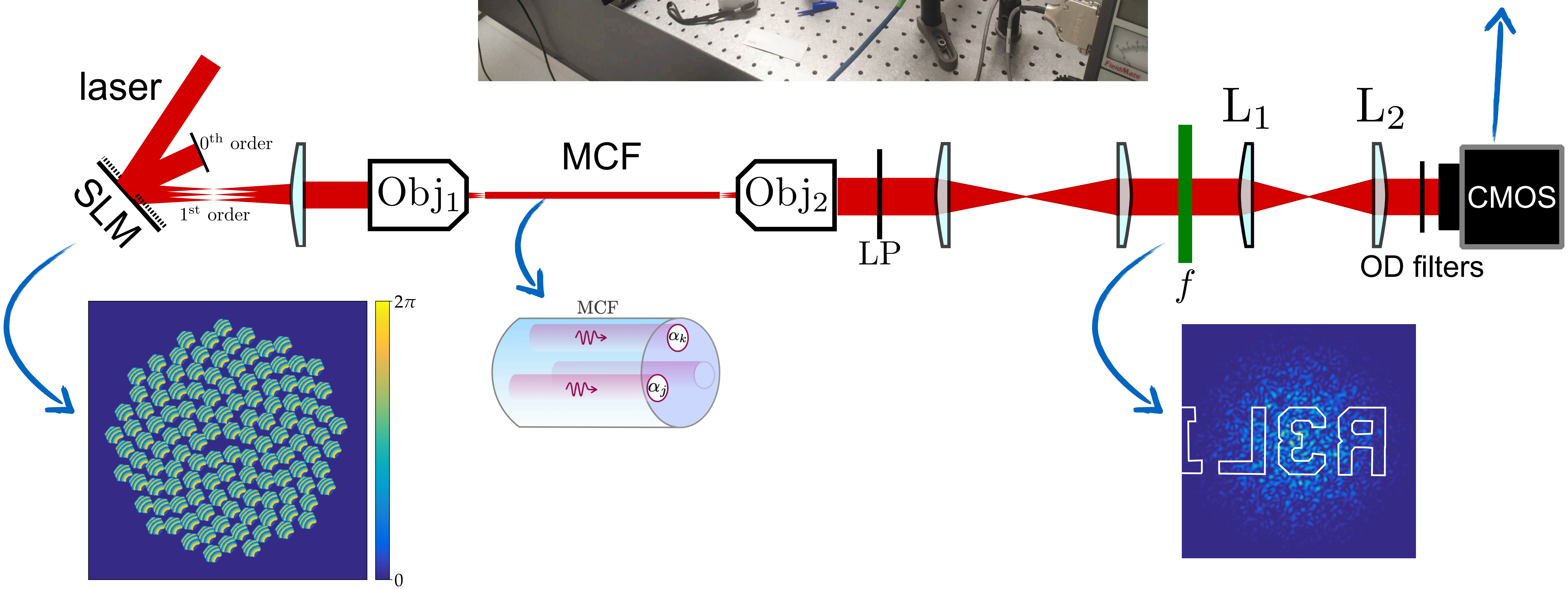




Experiments (in Institut Fresnel, France)



Single pixel
=
Pixel summation



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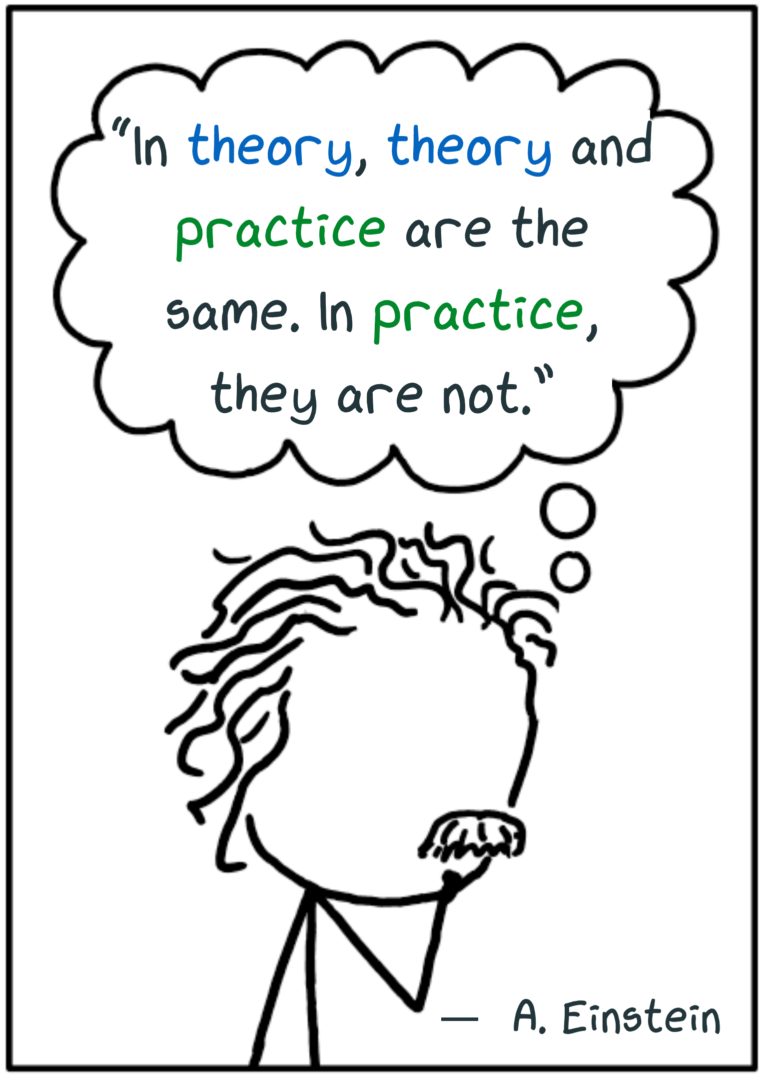
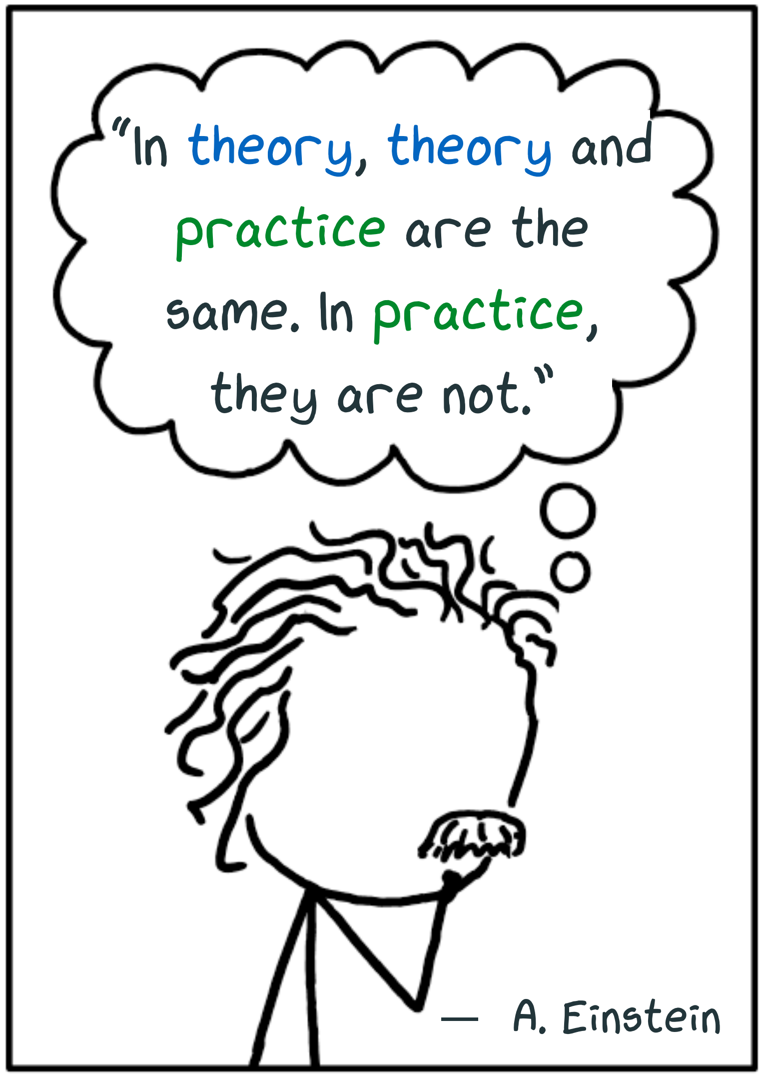


(Adapted from xkcd #1233)

Experiments (in Institut Fresnel, France)



(Adapted from xkcd #1233)

Special points of attention:

- MCF must be calibrated
- MCF system in transmission mode
- Speckle calibration (system imperfections)
  e.g., $\neq$ core radius, locations, ...

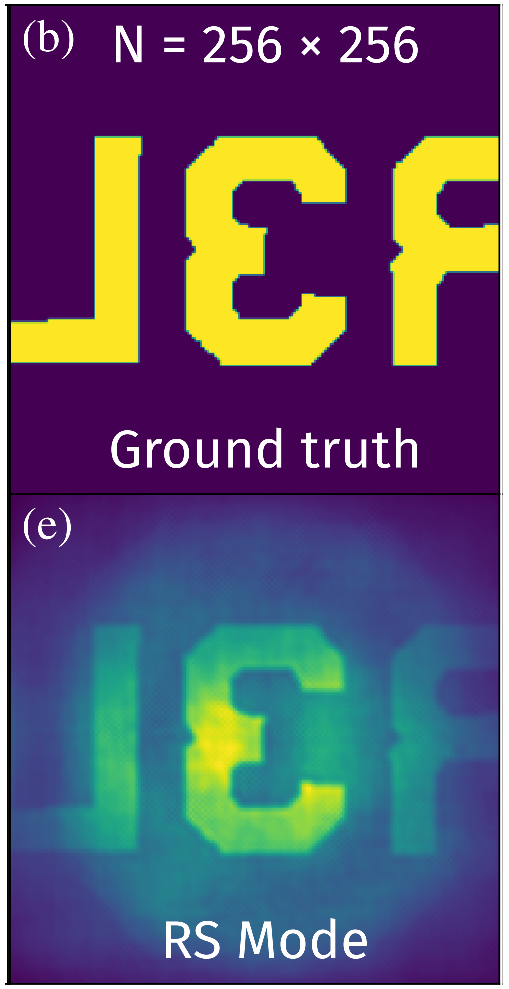
Reconstruction method: TV regulariser

$$\tilde{\boldsymbol{f}} = \arg\min_{\boldsymbol{f}} \ \tfrac{1}{2}\|\boldsymbol{y}^{\mathrm{c}} - \boldsymbol{\mathcal{B}}(\boldsymbol{f})\|_2^2 + \rho\|\boldsymbol{f}\|_{\mathrm{TV}} \ \text{s.t.}\ \boldsymbol{f} \geq 0,$$

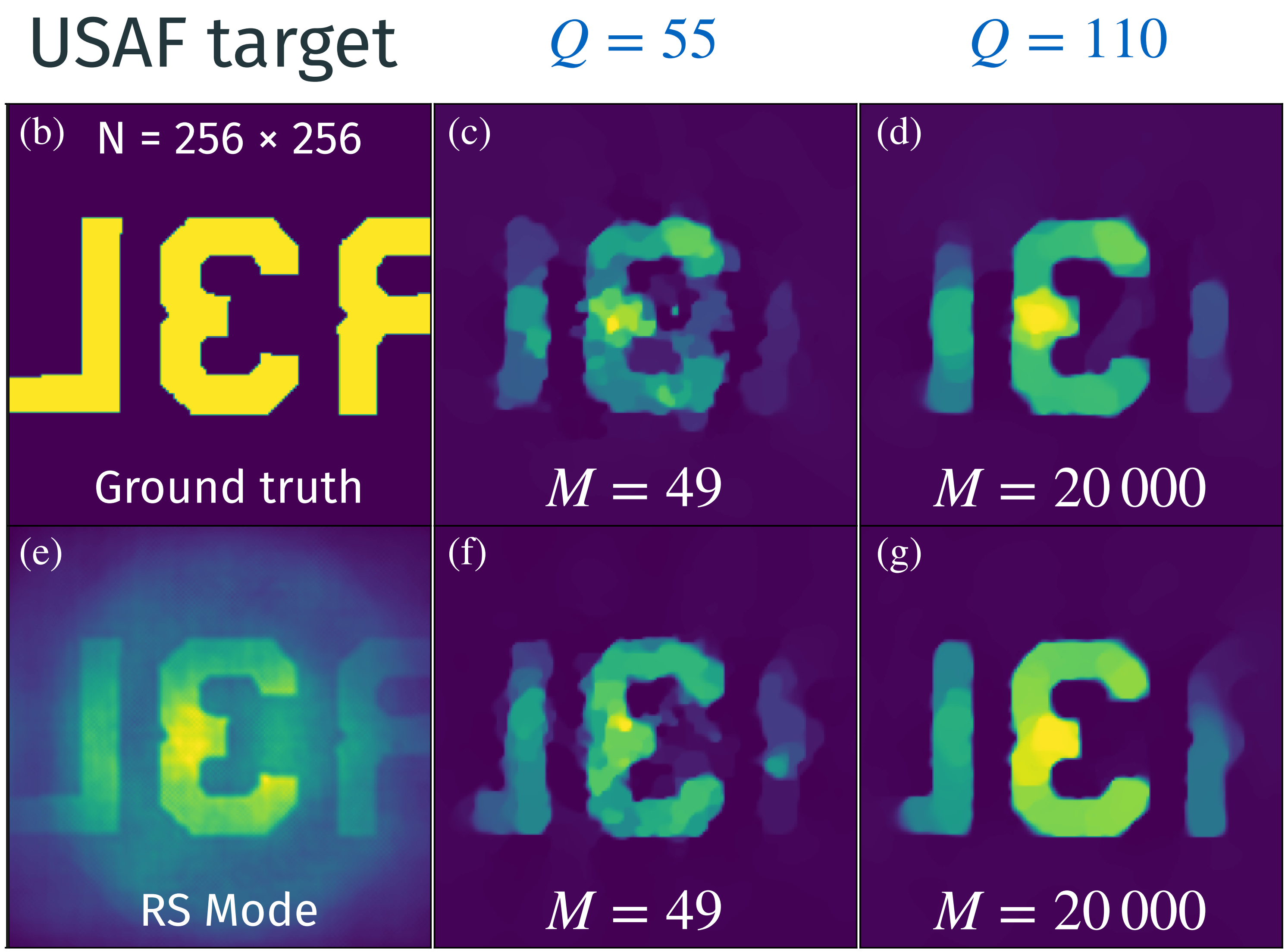
$\boldsymbol{\mathcal{B}}$ → ROP + interfero

ρ → Set empirically

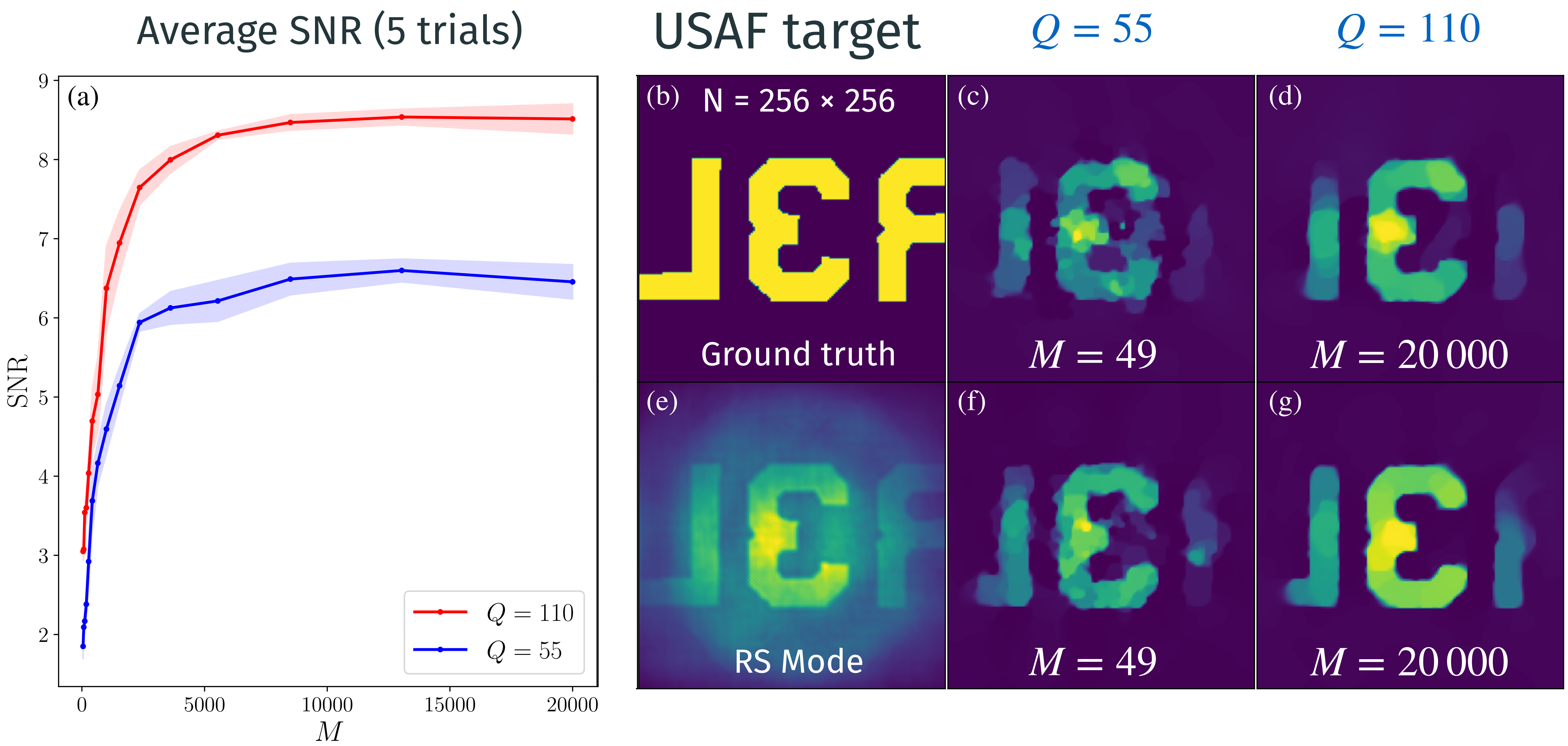
USAF target



Experiments (in Institut Fresnel, France)



Experiments (in Institut Fresnel, France)



To conclude ...

Take away messages:

- Fluorescent compressive speckle imaging (with MCF) follows an interferometric sensing model;
- This model amounts to "rank-one projecting" an interferometric matrix.
- This matrix has low-complexity

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- Fluorescent compressive speckle imaging (with MCF) follows an interferometric sensing model;
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Open questions:

- Using more advanced sparsity models:
  e.g., sparsity in levels + wavelets, weighted sparsity
- Optimization of core arrangements
- Extension to 3D imaging
- Data-driven calibration and reconstruction

Thank you for your attention!

Related bibliography:

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