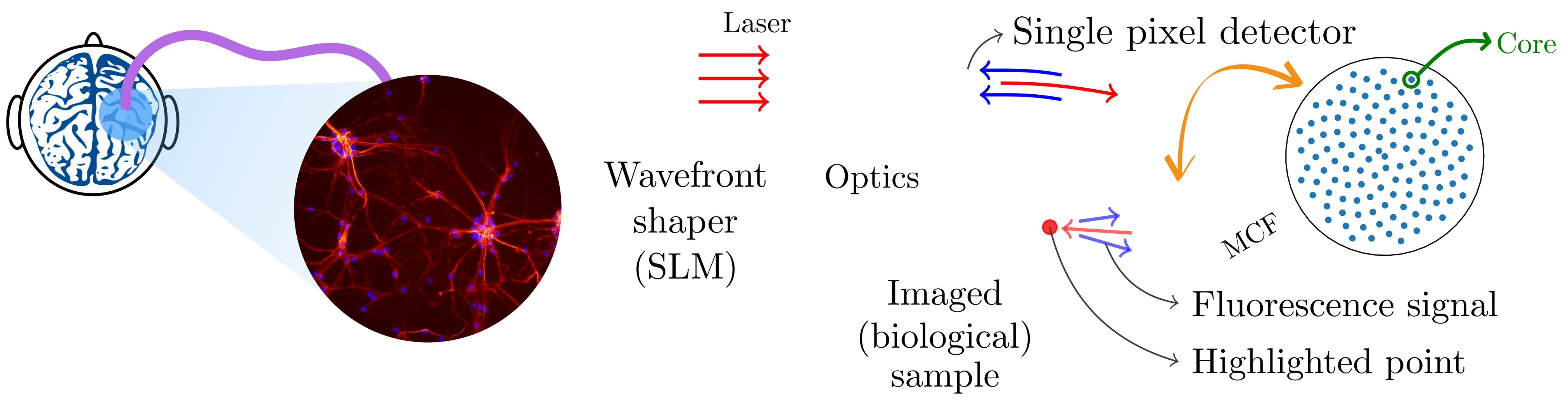
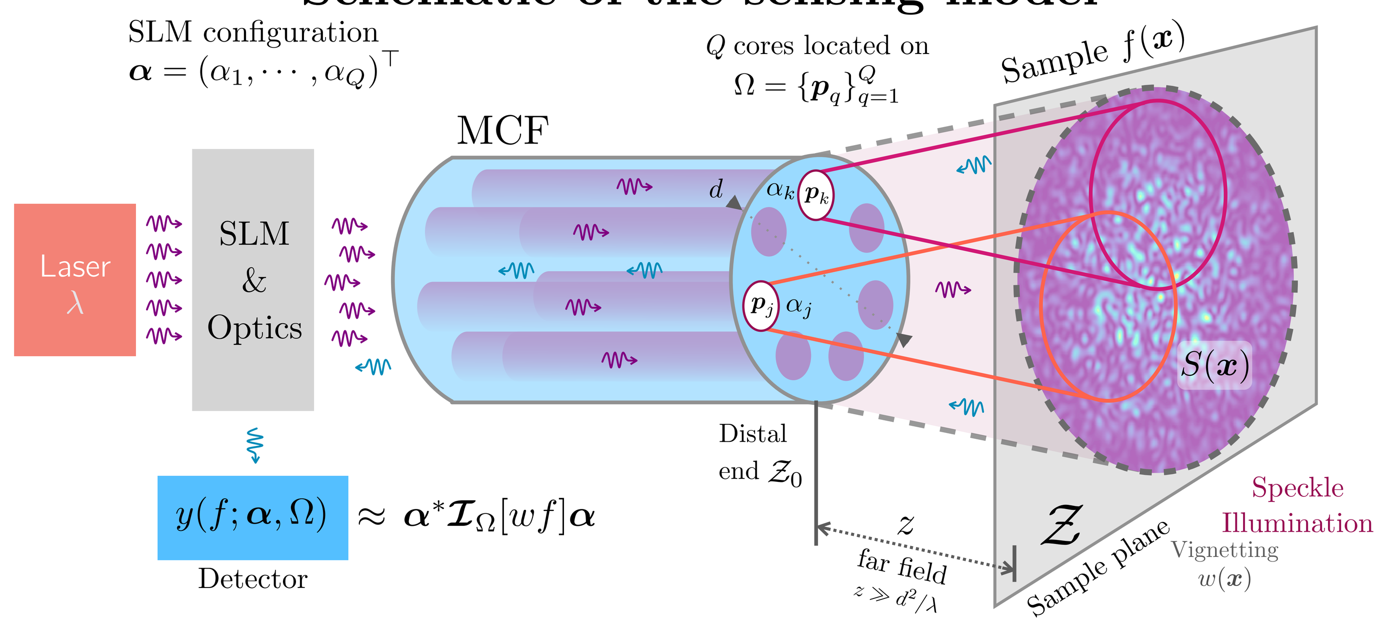
An Interferometric view of Speckle Imaging

1. Context : Towards Lenslesss Endoscopy

Case of study [1]



Schematic of the sensing model



How to see neurons firing?
- Image at microscale, in depth.
- With the least invasivity.

Solution:
→ Lensless endoscopy!

- SLM controls the light injected in each core of the MCF.
- The cores are aranged in a Golden Fermat's spiral [1].
- The light reflected/reemitted by the sample is integrated in a single-pixel detector.

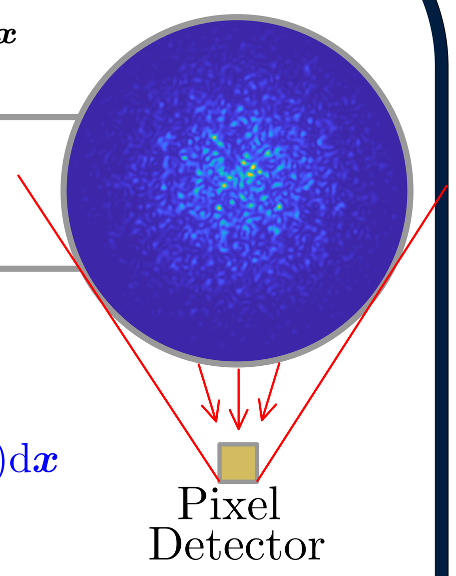
- The cores define a set Ω critical for the sensing ⇔ speckle illumination.
- Far-field assumption: MCF diameter $\ll$ Sample-Distal End distance.
- Field-of-view ⇔ speckle illumination diameter ⇔ core mode field.

2. Interferometric sensing model

Complex light wavefield emitted by core q
$$E_z(\boldsymbol{x},\boldsymbol{p}_q) = \alpha_q e^{\mathrm{i}kz} e^{\frac{\mathrm{i}k\|\boldsymbol{x}\|_2^2}{2z}} \sqrt{w(\boldsymbol{x})} e^{\frac{-2\pi\mathrm{i}}{\lambda z}\boldsymbol{p}_q^\top \boldsymbol{x}}$$

M different illuminating speckles
$$S(\boldsymbol{x};\boldsymbol{\alpha}) \approx w(\boldsymbol{x}) \left|\sum_{q=1}^{Q} \alpha_q e^{\frac{-2\pi\mathrm{i}}{\lambda z}\boldsymbol{p}_q^\top \boldsymbol{x}}\right|^2$$

Single speckle illumination = intensity of Q coded core wavefields
$$y(f;\boldsymbol{\alpha},\Omega) = \int_{\mathbb{R}^2} S(\boldsymbol{x};\boldsymbol{\alpha}) f(\boldsymbol{x}) \mathrm{d}\boldsymbol{x}$$
$$= \sum_{j,k=1}^{Q} \alpha_j^* \alpha_k \int_{\mathbb{R}^2} e^{\frac{2\pi\mathrm{i}}{\lambda z}(\boldsymbol{p}_j-\boldsymbol{p}_k)^\top \boldsymbol{x}} w(\boldsymbol{x}) f(\boldsymbol{x}) \mathrm{d}\boldsymbol{x}$$
$$= \boldsymbol{\alpha}^* \mathcal{I}_\Omega[wf]\boldsymbol{\alpha} \in \mathbb{R}_+ \Rightarrow \text{linear in } f$$



Collecting M measurements ⇔ M SLM codes $\{\boldsymbol{\alpha}_m\}_{m=1}^{M}$
$$\{y_m\}_{m=1}^{M} = \boldsymbol{y} = \boldsymbol{\mathcal{A}} \circ \mathcal{I}_\Omega[\bar{f}] = \boldsymbol{\mathcal{B}} f$$

Two-step measurement

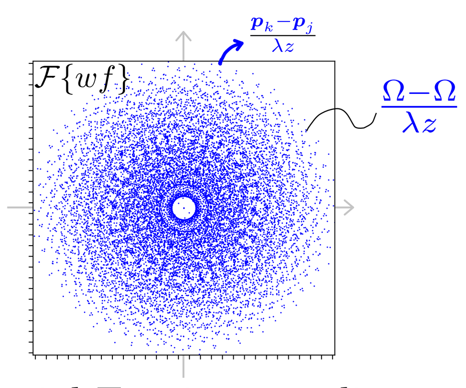
Rank-One Projection (ROP)
$$(\boldsymbol{\mathcal{A}}[\boldsymbol{H}])_m := \boldsymbol{\alpha}_m^* \boldsymbol{H} \boldsymbol{\alpha}_m$$

Interferometric matrix
$$\mathcal{I}_\Omega[\bar{f}] = \mathcal{F}\{\bar{f}\}\left[\tfrac{\boldsymbol{p}_k - \boldsymbol{p}_j}{\lambda z}\right]$$

Asymptotic problem $\mathbb{E}[\boldsymbol{\mathcal{A}}^*\boldsymbol{\mathcal{A}}\mathcal{I}_\Omega] \neq \mathcal{I}_\Omega$ if $\operatorname{tr}\mathcal{I}_\Omega \neq 0$

Solution → Diagless ROP

$$(\boldsymbol{\mathcal{A}}_{\mathrm{dl}}[\boldsymbol{H}])_m := \boldsymbol{\alpha}_m^* (\boldsymbol{H} - \operatorname{diag} \boldsymbol{H}) \boldsymbol{\alpha}_m$$
⇔ debiasing the measurements $\boldsymbol{y}$



Partial Fourier sampling

3. Interferometric structural models

Low complexity model (LCM) in f	LCM in $\mathcal{I}_\Omega$
Spike model	Low-rankness
$\bar{f}(\boldsymbol{x}) = \sum_{i=1}^{K} \rho_i \delta(\boldsymbol{x} - \boldsymbol{x}_i)$ for $K \ll Q$	$\mathcal{I}_\Omega[\bar{f}] = \sum_i \rho_i \boldsymbol{u}(\boldsymbol{x}_i)\boldsymbol{u}^*(\boldsymbol{x}_i)$, $\boldsymbol{u}(\boldsymbol{x})_j := e^{\mathrm{i}2\pi\boldsymbol{p}_j^\top \boldsymbol{x}}$
Sparsity in $\{\psi\}_{k=1}^{d}$	Sparsity in $\{\mathcal{I}_\Omega[\psi_k]\}_{k=1}^{d}$
$\bar{f}(\boldsymbol{x}) = \sum_{k=1}^{d} \rho_k \psi_k(\boldsymbol{x})$ with $\|\boldsymbol{\rho}\|_0 = K \ll d$	$\mathcal{I}_\Omega[\bar{f}] = \sum_{k \mid \rho_k \neq 0}^{K} \rho_k \mathcal{I}_\Omega[\psi_k]$

Remark: $\mathcal{I}_\Omega$ is almost circulant ⇒ $\mathcal{I}_\Omega = \boldsymbol{C}^*\boldsymbol{\mathcal{J}}\boldsymbol{C}$ with $\boldsymbol{\mathcal{J}}$ a circulant matrix.
⇒ Forward model = autocorrelative model to reduce computational cost
$$y_m = \boldsymbol{\alpha}_m^* \boldsymbol{C}^* \boldsymbol{\mathcal{J}} \boldsymbol{C} \boldsymbol{\alpha}_m = \boldsymbol{\beta}_m^* \boldsymbol{\mathcal{J}} \boldsymbol{\beta}_m = (\boldsymbol{\beta}_m * \overline{\boldsymbol{\beta}}_m) \boldsymbol{F} \bar{\boldsymbol{f}}$$

1 Reconstruction algorithm: given $\boldsymbol{\mathcal{B}} = \boldsymbol{\mathcal{A}} \circ \mathcal{T} \circ \boldsymbol{F}$

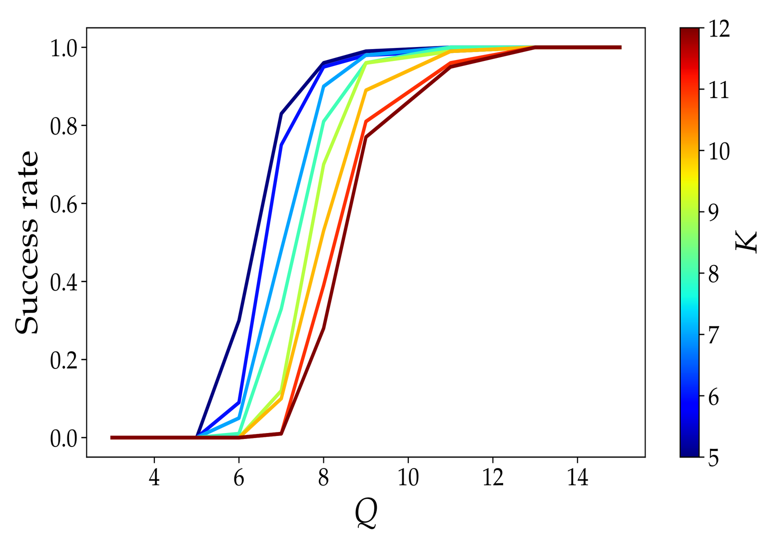
- If f is discretised, and sparse in pixels, hope to solve the inverse problem with
$$\hat{\boldsymbol{f}} \in \arg\min_{\boldsymbol{u}} \|\boldsymbol{u}\|_1 \ \text{ s.t. } \big\|\underbrace{\boldsymbol{\mathcal{B}}(\boldsymbol{f}) + \boldsymbol{n}}_{\boldsymbol{y}} - \boldsymbol{\mathcal{B}}(\boldsymbol{u})\big\|_1 \leqslant \epsilon \qquad (\mathrm{BPDN}_{\ell_1})$$

2 ℓ_2/ℓ_1 instance optimality of BPDN_{ℓ_1}: given $K > 2k$, if:
- for $k' \in \{K, K+k\}$, we have $\alpha_{k'}\|u\|_2 \leq \|\boldsymbol{\mathcal{B}}(u)\|_1 \leq \beta_{k'}\|u\|_2$, $\forall k'$-sparse u
- and $\frac{1}{\sqrt{2}}\alpha_{k+K} - \beta_K \frac{\sqrt{k}}{\sqrt{K}} > \gamma > 0$,

Then
$$\|\boldsymbol{f} - \hat{\boldsymbol{f}}\|_2 \leq \underbrace{C\frac{\|\boldsymbol{f} - \boldsymbol{f}_K\|_1}{\sqrt{K}}}_{\text{Model error}} + \underbrace{D\frac{\epsilon}{m}}_{\text{Noise error}}$$

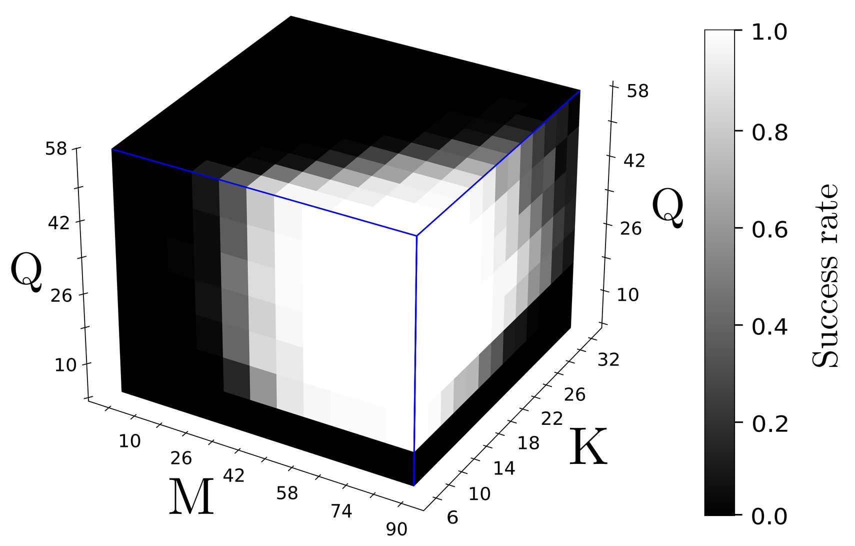
4. Numerical experiments

Recover 1-D signal solving $\hat{f} = \arg\min_f \frac{1}{2}\|\boldsymbol{y} - \boldsymbol{\mathcal{B}} f\|_2^2 + \lambda\|f\|_1$, with $\begin{cases} p_q \in \mathbb{R} \sim \mathcal{U}[0,255] \\ \boldsymbol{\alpha} \underset{\text{i.i.d.}}{\sim} \mathcal{N}(\boldsymbol{0}, \boldsymbol{I}_Q) \end{cases}$



Transition curves obtained with $M = 100$ ROPs, ensures recovery of $\mathcal{I}_\Omega$.

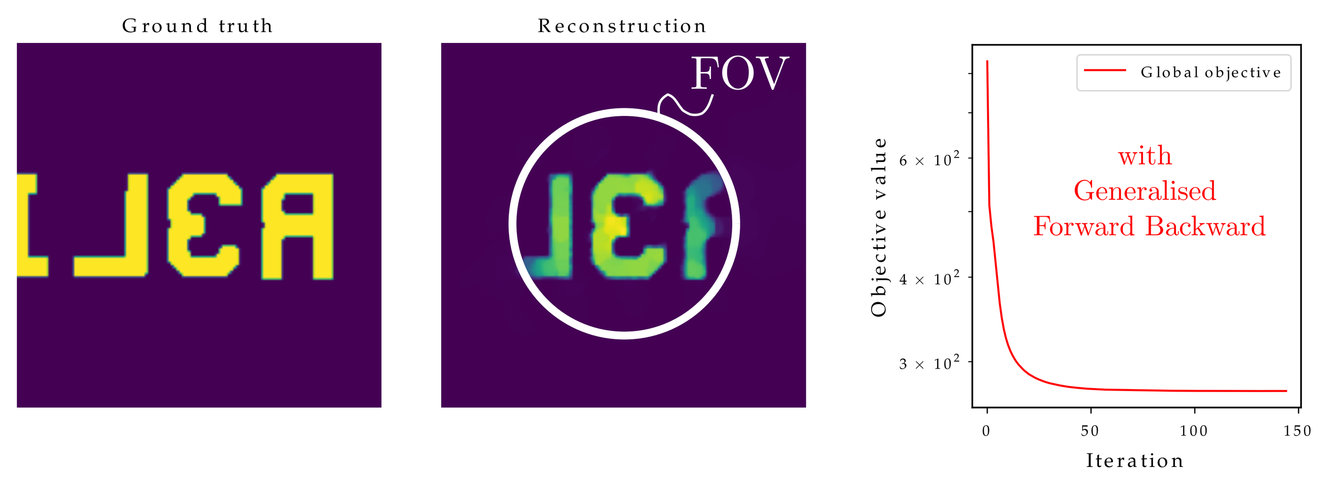
If $K \nearrow$ ⇒ transition point (in Q) $\nearrow$.



Transition "cube".
M ROPs of $\mathcal{I}_\Omega[wf] \in \mathbb{C}^{Q \times Q}$ for K-sparse f.
60 trials/pixel, success if SNR$\geq$ 40dB.

5. Real experiments

Recover piecewise constant object solving
$$\hat{f} = \arg\min_f \ \tfrac{1}{2}\|\boldsymbol{y} - \boldsymbol{\mathcal{B}} f\|_2^2 + \lambda\|f\|_{\mathrm{TV}} \ \text{ s.t. } \ f \geq 0$$



- Reconstruction limited by FOV
  ⇒ compute SNR in FWHM of envelope region.
- Both GT and estimate intensities are renormalized to [0,1].
- SNR: 11dB.

References

[1] S. Sivankutty, V. Tsvirkun, G. Bouwmans, D. Kogan, D. Oron, E. R. An- dresen, and H. Rigneault. Extended field-of-view in a lensless endoscope using an aperiodic multicore fiber, Optics Letters, 41 (2016), p. 3531

[2] E. Candès, J. K. Romberg, and T. Tao. Stable signal recovery from incomplete and inaccurate measurements. Comm. on Pure and Applied Math., LIX:1207–1223, 2006.

[3] S. Guérit, S. Sivankutty, J. Lee, H. Rigneault, L. Jacques. Compressive Imaging Through Optical Fiber with Partial Speckle Scanning. SIAM Journal on Imaging Sciences, 2021.

[4] Y. Chen, Y. Chi, and A. J. Goldsmith. Exact and stable covariance estimation from quadratic sampling via convex programming. IEEE Transactions on Information Theory, 61(7):4034–4059, 2015.

[5] T. Cai, A. Zhang, et al. Rop: Matrix recovery via rank-one projections. Annals of Statistics, 43(1):102–138, 2015.

Conclusion

- Forward problem close to phase retrieval, interferometric model, but still linear in f.
- LCM in f yields LCM in $\mathcal{I}_\Omega$.
- Potential for computational cost reduction.
- Theoretical guarantees.
- First experimental results ⇒ Proof of concept.

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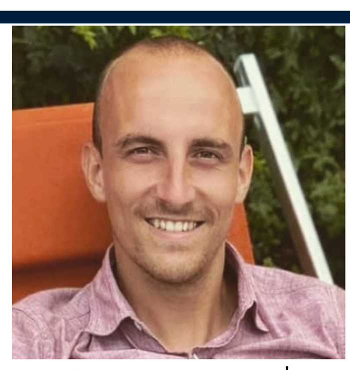

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Acknowledgment

LJ and OL are funded by Belgian National Science Foundation (F.R.S.-FNRS).
Part of this research is funded by the Fonds de la Recherche Scientifique– FNRS under Grant no T. 0136.20 (Learn2Sense).